Physical models for plasmas II

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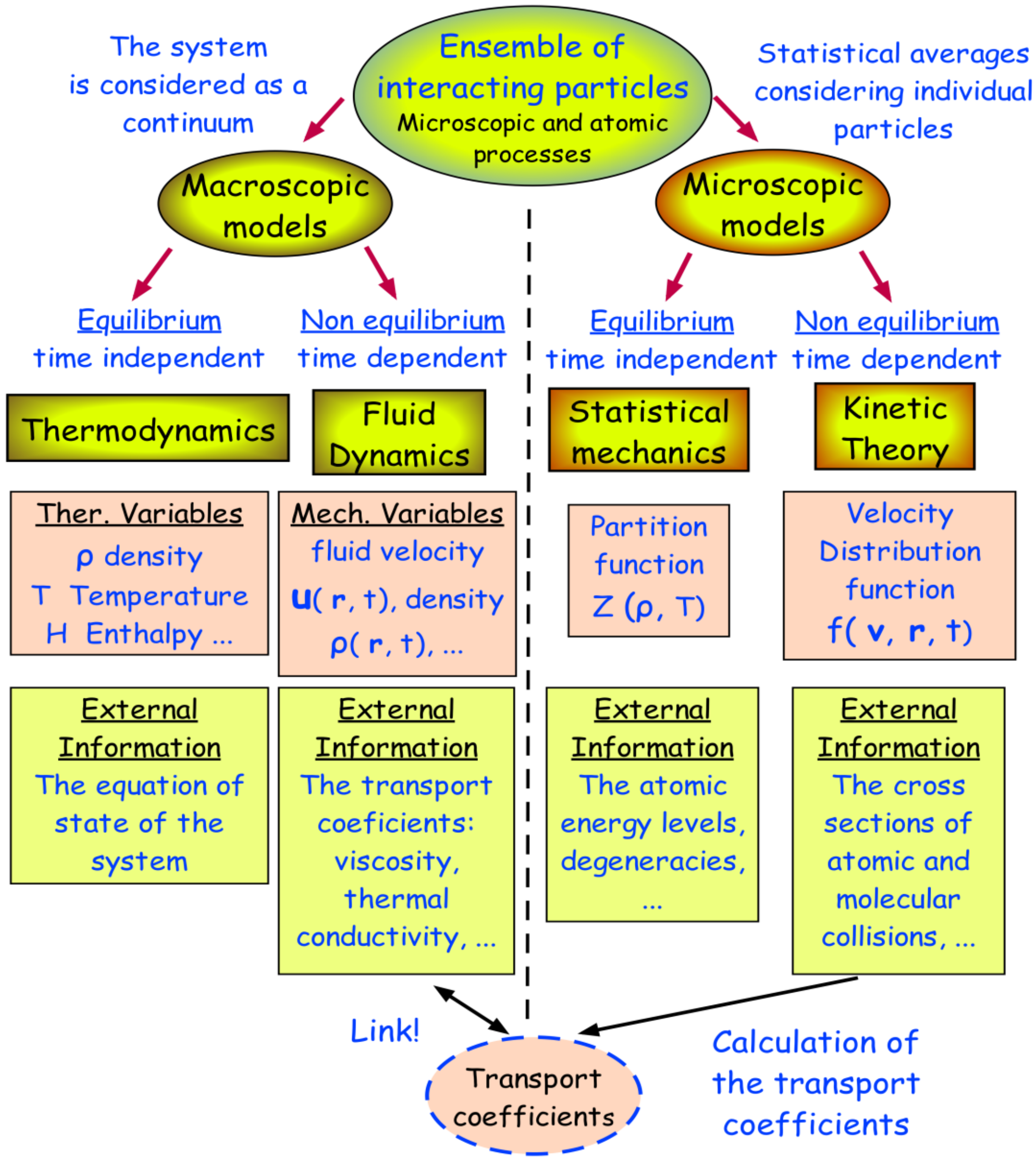
Physical models, ...



Plasma Kinetic Theory

Plasma kinetic theory **basis**: description of the dynamics of **neutral gases**, i.e., a velocity distribution function for the number of particles at any given phase-space point having a particular set of component velocities:

integro-differential equations appear , as the well-known ***Boltzmann equation***

Applied to plasmas: Kinetic theory gives the fundamental description of a plasma state, specially for non-equilibrium plasmas.

For quasi-equilibrium plasmas, **simplified fluid equations can be derived** for time evolution of measurable **macroscopic quantities** (density, fluid velocity, temperature) **as suitable averages** of the velocity distribution function.

Fundamental parameters: density, Energy (temperature) , pressure (stress tensor) and heat flux.

e.g. For neutral gases in thermal equilibrium $p = n_0 k_B T$; *Does it hold for plasmas?*

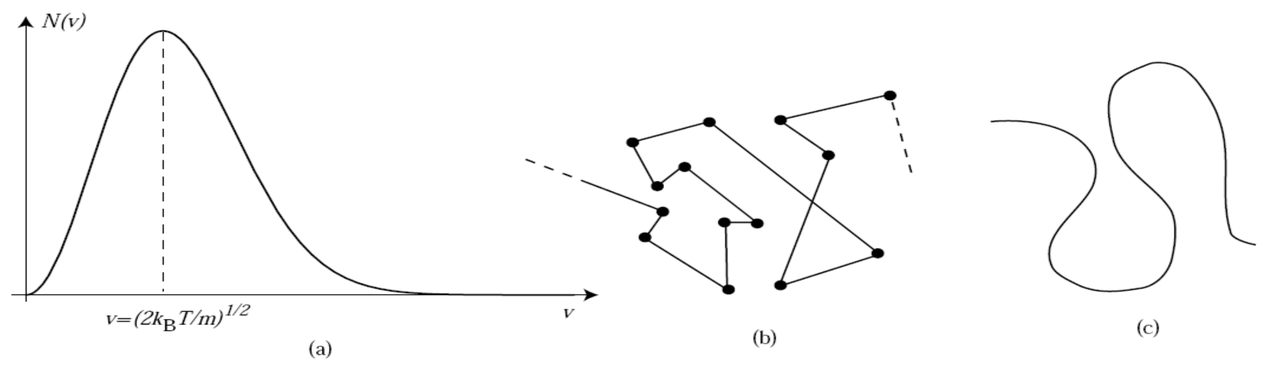
Plasma Kinetic Theory

For a neutral gas in thermal equilibrium at temperature *T*, the individual molecules have a great variety of velocity and kinetic energy values from zero to very large values.

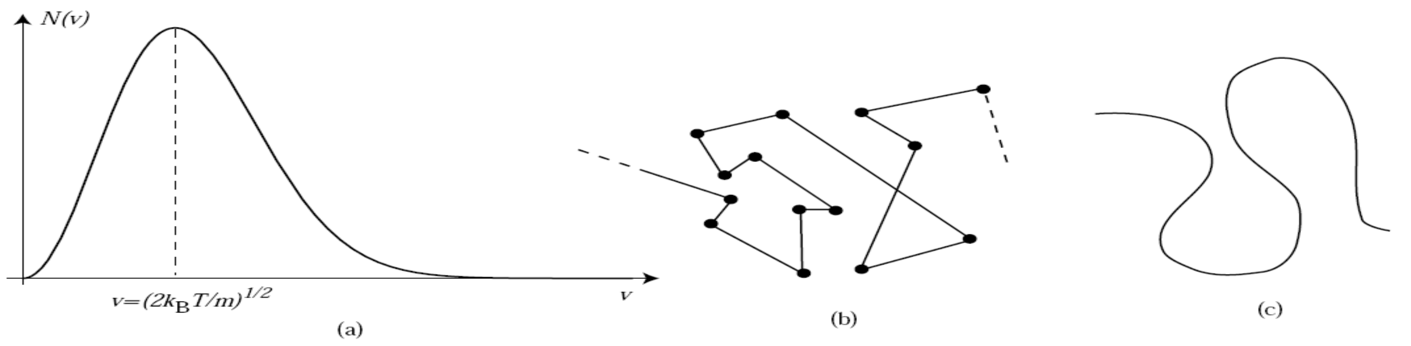
In Statistical Mechanics, a first approach firstly given by J. C. Maxwell, *the distribution fuinction* accounts for the number of molecules with speeds in the range between v and $v + \Delta v$ is given by

$$N(v) = Av^2 e^{-mv^2/(2k_B T)} \Delta v$$

$$N_0 = \int_0^\infty Av^2 e^{-mv^2/(2k_B T)} dv \quad \longrightarrow \quad A = N_0 \left[\frac{2}{\pi} \left(\frac{m}{k_B T} \right)^3 \right]^{1/2}$$



Plasma Kinetic Theory (neutral gas vs plasma) Differences:



Thermal equilibrium: $v = \sqrt{k_B T/m}$ is the most probable speed

(a) Maxwellian distribution.

(b) Random (zig-zag) motion of a neutral gas molecule by short range instantaneous interaction forces (elastic collisions, zig-zag path , ***mean free path***, generally denoted by λ)

(c) Trajectory (smooth path now !!) of a charged particle in a plasma (binary interaction via Coulomb force, realtively large-range interaction force)

Aim: to extend the Kinetic theory of gases, as a basis, to charged particle gas, although *different relaxation times* to reach the equilibrium have to be considered, many concepts usually used for gases are discussed for plasma.

Plasma Kinetic Theory (neutral gas vs plasma)

But, Plasma **Kinetic treatment is possible, however,** there **are great differences** , mainly related to the so-called 'collisions' (nature of the interaction) between particles.

In neutral gases: length scales of the system usually much larger than *λ;*
the time scales involved are much longer than mean-time between collisions *τ : quick* thermal equilibrium is reached for almost all of macroscopic variables (density, internal energy …)

Neutral particles: interact in a **spatial (and time) short-range** of intense forces (e.g. Maxwell *hard spheres* model, typical zig-zag)

Plasmas: the interaction via **long range** (and weak) Coulomb forces, the particles do not experience instantaneous "contact" collisions :
smooth random motion emerges in a average *field E (figures above)*

It is now important to distinguish interaction ***in/out*** *a Debye Sphere,* ***microscopic fields*** *simulated by collisional effects dominate in a Debye Sphere****, whereas macroscopic*** *fields* (contribution ***out*** of Debye Sphere) *also enter as a response of the collective effects, intrinsic in any plasma.*

Kinetic description: The distribution function

Its meaning: It is considered the number of particles in a 6-D point space $(\mathbf{r}, \mathbf{v})$ **of volume** $d\mathbf{r}\, d\mathbf{v}$ as

$$f(\mathbf{r}, \mathbf{v}, t)\, d\mathbf{r}\, d\mathbf{v}$$

Any number $n(\mathbf{r}, t)\, d\mathbf{r}$ of particles in 3-D real spatial volume $d\mathbf{r}$ is set in a point of velocity space $\mathbf{v}$ as it were "*a particle*" in the volume $d\mathbf{r}$ at instant *t.*

Particles can pass the boundary of the volume $d\mathbf{r}$ **:e.g.** particle fluxes due to collisions. Particles at any point are also "distributed" in velocity.

As done for the the particle density in a point of the configuration space $\mathbf{r}$, a density of $n\, d\mathbf{r}$ points in velocity space is defined being proportional to volume element $d\mathbf{r}$ and function *of* $\mathbf{r}$, t and $\mathbf{v}$ *as* $f(\mathbf{r}, \mathbf{v}, t)\, d\mathbf{r}$.

The number of particles in the volume d***r*** with velocities lying between $\mathbf{v}$ and $\mathbf{v} + d\mathbf{v}$ is $f(\mathbf{r}, \mathbf{v}, t)\, d\mathbf{r}\, d\mathbf{v}$. ***Finally:***

$f(\mathbf{r}, \mathbf{v}, t)$ is the ***velocity distribution function,*** it can be understood as a probability density of points in the 6-D ***r-v*** phase space.

A very general formulation is possible knowing the time evolution of f, valid for Inhomogeneous, anisotropic and non-equilibrium plasmas. E.g.

$$f_{\text{beam}}(\mathbf{r}, \mathbf{v}, t) = f_{\text{beam}}(\mathbf{v}) = N_0\, \delta(v_x - v_0)\, \delta(v_y)\, \delta(v_z) \qquad , \qquad f_{\text{shell}}(\mathbf{r}, \mathbf{v}, t) = f_{\text{shell}}(\mathbf{v}) = A\, \delta(v - v_0)$$

The kinetic description of plasmas ...

So that, the key concept is the non-equilibrium velocity distribution for each plasma species ,which introduces a probabilistic description:

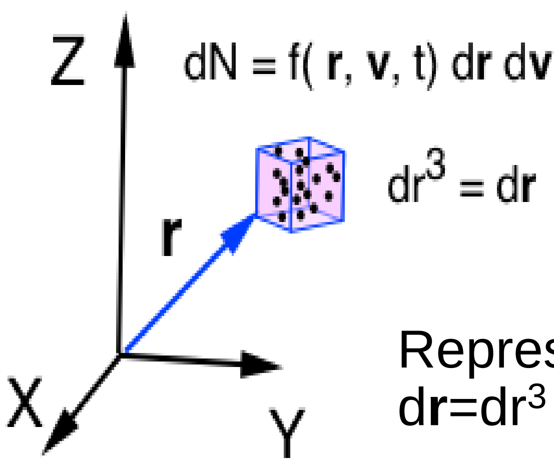
$$f_\alpha(\mathbf{v}, \mathbf{r}, t) \qquad \alpha = i, e, a$$

$$dn_\alpha = f_\alpha(\mathbf{v}, \mathbf{r}, t)\, d\mathbf{v}$$

The number density of particles with velocities in the range (***v***, ***v***+*d**v***) at the instant *t*

$$dN_\alpha = f_\alpha(\mathbf{v}, \mathbf{r}, t)\, d\mathbf{v}\, d\mathbf{r}$$

The number of particles within the volume *d**r*** and velocities in the range (***v***, ***v***+*d**v***) at the instant t



The number density: $n_\alpha(\mathbf{r}, t) = \int_{-\infty}^{+\infty} f_\alpha(\mathbf{v}, \mathbf{r}, t)\, d\mathbf{v}$

$$\hat{f}_\alpha(\mathbf{v}, \mathbf{r}, t) = \frac{f_\alpha(\mathbf{v}, \mathbf{r}, t)}{n_\alpha(\mathbf{r}, t)} \qquad 1 = \int_{-\infty}^{+\infty} \hat{f}_\alpha(\mathbf{v}, \mathbf{r}, t)\, d\mathbf{v}$$

Represents the probability of finding a particle within the volume d**r**=dr^3 with velocity in the range (***v***, ***v***+*d**v***) at the instant t

Normalization: See Chen, page 226; Eqs. 7.2 and 7.3 and Lecture notes Eqs. 5.1 and 5.3

The physical measurable magnitudes are averages:

Averages: the particle flux, kinetic energy by unit volume $\alpha = i, e, a$

$$\begin{cases} d\mathbf{\Gamma}_\alpha = \mathbf{v}\, dn_\alpha = \mathbf{v}\, f_\alpha(\mathbf{v}, \mathbf{r}, t)\, d\mathbf{v} \\ de_k = \dfrac{m_\alpha v^2}{2}\, dn_\alpha = \dfrac{m_\alpha v^2}{2} f_\alpha(\mathbf{v}, \mathbf{r}, t)\, d\mathbf{v} \end{cases}$$

The *macroscopic particle flux* becomes,(notation $\int_{-\infty}^{\infty}$ means integration over whole **v** space)

$$\mathbf{\Gamma}_\alpha(\mathbf{r}, t) = n_\alpha(\mathbf{r}, t)\, \mathbf{u}_\alpha(\mathbf{r}, t) = \int_{-\infty}^{+\infty} \mathbf{v}\, f_\alpha(\mathbf{v}, \mathbf{r}, t)\, d\mathbf{v}$$

or equivalently,

$$\mathbf{\Gamma}_\alpha(\mathbf{r}, t) = n_\alpha(\mathbf{r}, t)\, \mathbf{u}_\alpha(\mathbf{r}, t) = n_\alpha(\mathbf{r}, t) \int_{-\infty}^{+\infty} \mathbf{v}\, \hat{f}_\alpha(\mathbf{v}, \mathbf{r}, t)\, d\mathbf{v}$$

This defines the average *macroscopic fluid velocity:*

$$\mathbf{u}_\alpha(\mathbf{r}, t) = \frac{1}{n_\alpha(\mathbf{r}, t)} \int_{-\infty}^{+\infty} \frac{m_\alpha v^2}{2} f_\alpha(\mathbf{v}, \mathbf{r}, t)\, d\mathbf{v}$$

The average kinetic energy is related with the *local internal energy:*

$$e_{k\alpha}(\mathbf{r}, t) = \int_{-\infty}^{+\infty} \frac{m_\alpha v^2}{2} f_\alpha(\mathbf{v}, \mathbf{r}, t)\, d\mathbf{v}$$

for the MB monoatomic gases, $e_{\alpha,k}(\mathbf{r}, t) = \dfrac{3}{2} k_B T_\alpha(\mathbf{r}, t)$ *Local Equilibrium!*

The time evolution: Boltzmann and Vlasov equations …

Atomic and molecular *collisions* control the *time evolution* of the velocity distribution function $f_\alpha(r, v, t)$ *for any plasma species*

$$\frac{df_\alpha}{dt} = \frac{\partial f_\alpha}{\partial t} + \sum_i \left(\frac{\partial f_\alpha}{\partial x_i} \frac{dx_i}{dt} \right) + \sum_i \left(\frac{\partial f_\alpha}{\partial v_i} \frac{dv_i}{dt} \right) = \left(\frac{\partial f_\alpha}{\partial t} \right)_{col.}$$

And using,

$$\nabla_{\mathbf{r}} \equiv \left(\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k} \right) \qquad \nabla_{\mathbf{v}} \equiv \left(\frac{\partial}{\partial v_x}\mathbf{i} + \frac{\partial}{\partial V_y}\mathbf{j} + \frac{\partial}{\partial v_z}\mathbf{k} \right)$$

$$\left.\begin{array}{l} \dfrac{df_\alpha}{dt} = \dfrac{\partial f_\alpha}{\partial t} + \mathbf{v}\cdot\nabla_{\mathbf{r}} f_\alpha + \mathbf{a}\cdot\nabla_{\mathbf{v}} f_\alpha = C(f_\alpha) \\ \mathbf{a} = \mathbf{F}/m_\alpha \qquad \mathbf{F} = \mathbf{F}_g + q_\alpha\,(\mathbf{E} + \mathbf{v}\wedge\mathbf{B}) \end{array}\right\} \quad \begin{array}{l} \mathbf{v}\cdot\nabla_{\mathbf{r}}\, f_\alpha = \nabla_{\mathbf{r}}\cdot(\mathbf{v}\, f_\alpha) \\ \mathbf{a}\cdot\nabla_{\mathbf{v}}\, f_\alpha = \nabla_{\mathbf{v}}\cdot(\mathbf{a}\, f_\alpha) \end{array}$$

With these manipulations we derive the *Boltzmann equation* for the time and spatial evolution of $f_\alpha(r, v, t)$ becomes,

$$\boxed{\frac{\partial f_\alpha}{\partial t} + \nabla_{\mathbf{r}}\cdot(\mathbf{v}\, f_\alpha) + \nabla_{\mathbf{v}}\cdot\left(\frac{\mathbf{F}}{m_\alpha} f_\alpha\right) = C(f_\alpha)} \Leftrightarrow \frac{d}{dt} f_\alpha = C(f_\alpha) = \sum_\beta C_{col}(f_\alpha, f_\beta)$$

$$\boxed{C(f_\alpha) = 0}$$

Vlasov equation:
the function $f_\alpha(r, v, t)$ is also constant of motion and remains unchanged by the time evolution

$$\boxed{\frac{df_\alpha}{dt} = \left(\frac{\partial f_\alpha}{\partial t} \right)_{col.} = 0}$$

E.g. The Boltzmann Equation (a collisional operator)

The different collisions operators describe the time evolution of $f_\alpha(\mathbf{r}, \mathbf{v}, t)$ under the microscopic particles collisions.

Simplest collision model : Krook model, a drastic simplification

The Boltzmann operator applies to *binary collisions*,

$$\left(\frac{\partial f_\alpha}{\partial t}\right)_{col.} = C_s(f_\alpha) = \sum_\beta C(f_\alpha, f_\beta)$$

$$\boxed{C(f_\alpha, f_\beta) = \int\int (f'_\alpha f'_\beta - f_\alpha f_\beta)\, |\mathbf{g}|\, \sigma_{\alpha,\beta}(g,,\theta)\, d\Omega\, d\mathbf{v}_\beta}$$

$$|\mathbf{g}| = |\mathbf{v}_\alpha - \mathbf{v}_\beta| = |\mathbf{v}'_\alpha - \mathbf{v}'_\beta| = |\mathbf{g}'|$$

Neglecting collisions: Vlasov-Maxwell Equation work with only average fields.

Solvable problem?

Maxwell fields are responsible of **collective effects** (forces shielded **out**side Debye sphere)

$$\underbrace{\frac{\partial}{\partial t} f + (\mathbf{v}\cdot\nabla_{\mathbf{r}}) f + \left(\frac{\mathbf{F}}{m}\cdot\nabla_{\mathbf{v}}\right) f = 0}_{\textbf{Boltzmann}\ \text{Equation}} \quad ; \quad \underbrace{\frac{\partial f}{\partial t} + (\mathbf{v}\cdot\nabla_{\mathbf{r}})\, f + \left[(\mathbf{E} + \mathbf{v}\times\mathbf{B})\cdot\nabla_{\mathbf{v}}\right] f = 0}_{\textbf{Vlasov}\ \text{Equation (Boltzmann Equation for a Plasma)}}$$

$$\mathbf{J}(\mathbf{r},t) = \sum_i q_i N_i(\mathbf{r},t)\mathbf{u}_i(\mathbf{r},t) \quad ; \quad \rho(\mathbf{r},t) = \sum_i q_i N_i(\mathbf{r},t)$$

The Boltzmann collision operator ...

The different collisions operators describe the time evolution of $f_\alpha(r, v, t)$ under the microscopic particles collisions.The Boltzmann operator applies to *binary collisions*, whereas alternative formulations as the Fokker-Planck operator are employed when *several particles* are simultaneously involved, as in dense plasmas, **Coulomb collisions** control particle densities, momentum fluxes and energy relaxation processes.

More formally,

$$\left(\frac{\partial f_\alpha}{\partial t}\right)_{col.} = C_s(f_\alpha) = \sum_\beta C(f_\alpha, f_\beta)$$

The time evolution of the α particle distribution function

Takes place considering all binary collision processes with all others α ≠ β particle species present in the plasma.

We will concentrate into a **binary collision** approximation (*interaction between a single fixed test particle and another field particle of the same or different species*) as,

$$\left(\frac{\partial f_\alpha}{\partial t}\right)_{\alpha\beta} = C(f_\alpha, f_\beta)$$

that is, it gives rise to particular collisional process between the α and β species

Outline of the time evolution (see text) ...

We assume an isotropic energy distribution function in one dimension evolving from *g(E, t)* to *g(E, t+δt)* during *δt* and as in the figure, two energy intervals around *E* and *E'*

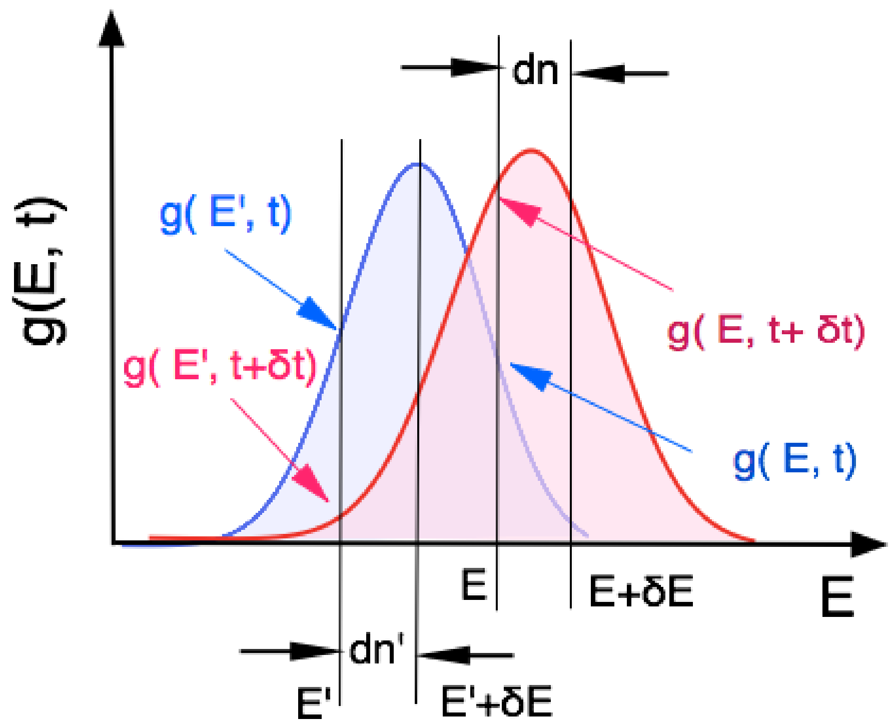
Within $\begin{cases} (E, E+\delta E) & g(E,t+\delta t) > g(E,t) \\ (E', E'+\delta E') & g(E',t+\delta t) < g(E',t) \end{cases}$ increases $dn = g(E,t+\delta t)\,dE$, decreases $dn' = g(E',t+\delta t)\,dE'$

In the energy interval *(E, t+δt)* we have,

$$\left(\frac{\delta g}{\delta t}\right) dE \simeq \frac{g(E,t+\delta t) - g(E,t)}{\delta t} dE > 0$$

Because the energy of particles changes in collisions, (δg/δt) has two contributions;

$$\left(\frac{\delta g}{\delta t}\right) = \left(\frac{\delta g}{\delta t}\right)^{+} - \left(\frac{\delta g}{\delta t}\right)^{-}$$



Particles with original energies outside (E, E+dE) *coming IN* this energy interval, from other energies accelerated or retarded during δt.

Particles with original energies within (E, E+dE) *coming OUT* from this energy interval towards others energies accelerated or retarded during δt.

More formal ...

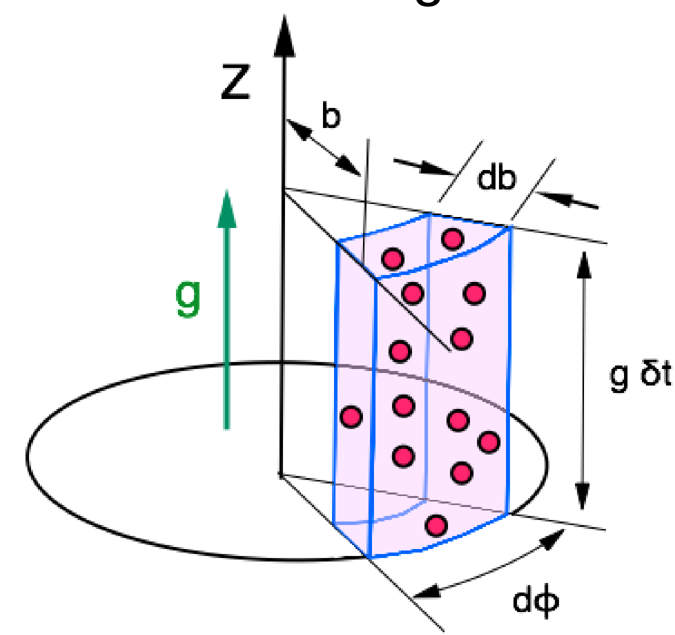
The rate of change of $f_\alpha(r, v, t)$ within the speed dv interval is,

$$\left(\frac{\partial f_\alpha}{\partial t}\right)_{\alpha,\beta} d\mathbf{v}_\alpha = \left[\left(\frac{\partial f_\alpha}{\partial t}\right)^{+}_{\alpha,\beta} - \left(\frac{\partial f_\alpha}{\partial t}\right)^{-}_{\alpha,\beta}\right] d\mathbf{v}_\alpha$$

In the frame where β particles are at rest for the stream of incoming α particles scattered by a SINGLE target particle β we have,

$$dn_\alpha = f_\alpha(\mathbf{r}, \mathbf{v}_\alpha, t)\, d\mathbf{v}_\alpha$$

Then, $\delta V = (|\mathbf{g}|\, \delta t) \times (b\, db) \times d\theta$ is the volume in shadow in the figure and the, number of particles inside, $dN_\alpha = dn_\alpha \times \delta V$



$$dN_\alpha = (f_\alpha(\mathbf{r}, \mathbf{v}_\alpha, t)\, d\mathbf{v}_\alpha) \times (|\mathbf{g}|\, \delta t) \times (b\, db) \times d\theta$$

By *direct collisions,* the number of α particles *removed from* the velocity interval $(v_\alpha, v_\alpha + dv_\alpha)$ is the number of encounters with β particles by volume during δt ,

$$dn_\beta \times dN_\alpha = dn_\beta \times dn_\alpha \times \delta V$$

and

hence,

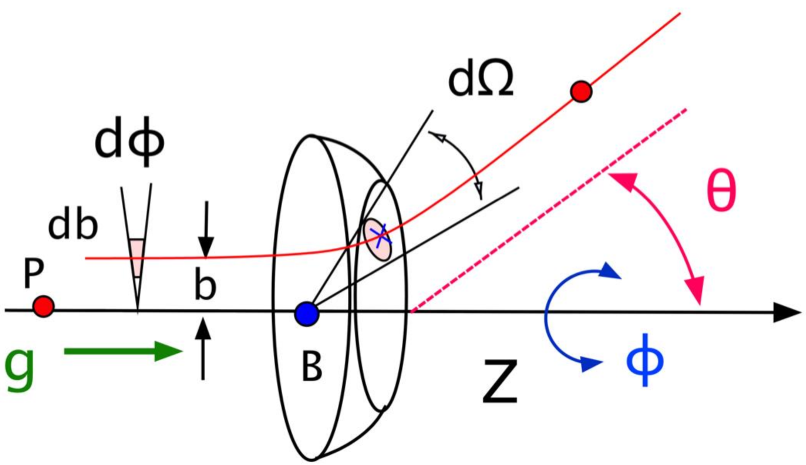
$$\left(\frac{\partial f_\alpha}{\partial t}\right)^{-}_{\alpha,\beta} d\mathbf{v}_\alpha\, \delta t = f_\alpha(\mathbf{r}, \mathbf{v}_\alpha, t) \times f_\beta(\mathbf{r}, \mathbf{v}_\beta, t) \times (|\mathbf{g}|\, \delta t) \times (b\, db\, d\theta)\, d\mathbf{v}_\alpha\, d\mathbf{v}_\beta$$

The Boltzmann collision operator ...

Equivalently, by *reverse collisions,* a number of *particles from* the velocity interval (v'_α, $v'_\alpha + dv'_\alpha$) *are added* to the interval (v_α, $v_\alpha + dv_\alpha$) and during the time δt ,

$$\left(\frac{\partial f_\alpha}{\partial t}\right)^{+}_{\alpha,\beta} d\mathbf{v}_\alpha\,\delta t = f_\alpha(\mathbf{r},\mathbf{v}'_\alpha,t)\times f_\beta(\mathbf{r},\mathbf{v}'_\beta,t)\times(|\mathbf{g}'|\,\delta t)\times(b\,db\,d\theta)\,d\mathbf{v}'_\alpha\,d\mathbf{v}'_\beta$$

We introduce at this point the here is introduce the collision differential cross section that characterizes the particular (α, β) collision process,



$$b\,db\,d\theta = \sigma_{\alpha,\beta}(g,\theta)\,d\theta\,d\phi = \sigma_{\alpha,\beta}(g,\theta)\,d\mathbf{\Omega}$$

The *Boltzmann collision operator* is for the evolution in time of $f_\alpha(r, v, t)$ under the (α, β) collisions is defined as the difference,

$$C(f_\alpha,f_\beta)\,d\mathbf{v}_\alpha = \left(\frac{\partial f_\alpha}{\partial t}\right)_{\alpha,\beta} d\mathbf{v}_\alpha$$

$$C(f_\alpha,f_\beta)\,d\mathbf{v}_\alpha = (f'_\alpha\,f'_\beta\,|\mathbf{g}'|\,d\mathbf{v}'_\alpha\,d\mathbf{v}'_\beta - f_\alpha\,f_\beta\,|\mathbf{g}|\,d\mathbf{v}_\alpha\,d\mathbf{v}_\beta)\,\sigma_{\alpha,\beta}(g,\theta)\,d\mathbf{\Omega}$$

For elastic collisions...

The particular properties of each collisional process simplify the previous expression. For *elastic collisions* momentum and energy are conserved and,

$$|\mathbf{g}| = |\mathbf{v}_\alpha - \mathbf{v}_\beta| = |\mathbf{v}'_\alpha - \mathbf{v}'_\beta| = |\mathbf{g}'|$$

also,

$$d\mathbf{v}_\alpha \, d\mathbf{v}_\beta = d\mathbf{v}'_\alpha \, d\mathbf{v}'_\beta$$

This simplifies the Boltzmann collision operator,

$$C(f_\alpha, f_\beta) = \int \int (f'_\alpha \, f'_\beta - f_\alpha \, f_\beta) \, |\mathbf{g}| \, \sigma_{\alpha,\beta}(g, , \theta) \, d\Omega \, d\mathbf{v}_\beta$$

When the distribution functions are known we might exactly calculate transport coefficients as the rates of collisions with momentum transfer, etc.

Intuitive physical meaning*: observe that the difference of products of f functions means a **double proportionality** accounting for the collisions due to the **number of encounters between particles** , both numbers are weighted by an effective cross-section and by the value of the relative velocity, g and g'.*

Applications beyond the Physics...

(12) **United States Patent**
Uenohara et al.

(10) **Patent No.: US 7,552,076 B1**
(45) **Date of Patent: Jun. 23, 2009**

(54) **SYSTEM FOR EVALUATING PRICE RISK OF FINANCIAL PRODUCT OR ITS FINANCIAL DERIVATIVE, DEALING SYSTEM AND RECORDED MEDIUM**

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See application file for complete search history.

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(57) **ABSTRACT**

A system for correctly evaluating price distribution and risk distribution for a financial product or its derivatives introduces a probability density function generated with a Boltzmann model at a higher accuracy than the Gaussian distribution for a probability density. The system has an initial value setup unit and an evaluation condition setup unit. Initial values include at least one of price, price change rate, and price change direction of a financial product. The evaluation conditions include at least time steps and a number of trials. A Boltzmann model analysis unit receives the initial values and the evaluation conditions, and repeats simulations of price fluctuation, based on the Boltzmann model using a Monte Carlo method. A velocity/direction distribution setup unit supplies probability distributions of the price, price change rate, and price change direction for the financial product to the Boltzmann model analysis unit. A random number generator for a Monte Carlo method is employed in the analysis by the Boltzmann model, and an output unit displays the analysis result. A dealing system applies the financial Boltzmann model to option pricing, and reproduces the characteristics of Leptokurcity and Fat-tail by a linear Boltzmann equation to define risk-neutral and unique probability measures. Consequently, option prices can be evaluated in a risk-neutral and unique manner, taking into account Leptokurcity and Fat-tail of a price change distribution.

The kinetic theory is an statistical approach that goes beyond the Physics. It also applies to financial analysis, etc.

Portrait of Ludwig Boltzmann and his grave in Zentralfriedhof, Vienna