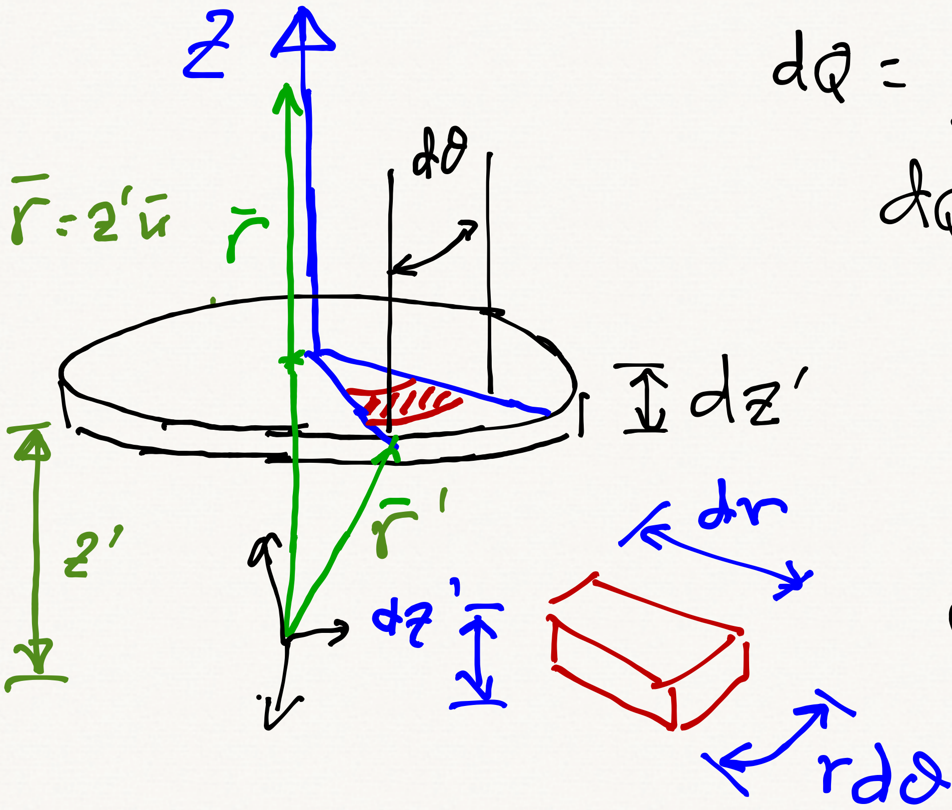
**Prob. 3.5**




$$dQ = \rho_0 \, dV$$

$$dQ = \rho_0 (r\,d\theta) \times (dr) \times dz'$$

$$\begin{cases} \vec{r} = z\vec{k} \\ \vec{r}\,' = r\,\vec{u}_r + z'\vec{k} \end{cases}$$

$$(\bar{r} - \bar{r}') = -r\bar{u}_r + (z - z')\vec{k}$$

$$|\bar{r} - \bar{r}'| = r^2 + (z - z')^2$$

El campo eléctrico tendrá a lo largo del eje dos componentes $d\bar{E} = dE_r + dE_z$ y $\cancel{dE_r} = 0$ por simetría.

$$d\bar{E} = \frac{1}{4\pi\epsilon_0} \frac{dQ}{[r^2 + (z - z')^2]^{3/2}} [-r\bar{u}_r + (z - z')\bar{k}]$$

La integración sobre $d\theta$ es directa

$$dE_z = \frac{1}{4\pi\epsilon_0} \rho_0 (2\pi) \frac{r(z - z')}{[r^2 + (z - z')^2]^{3/2}} \, dr\, dz'$$

e integramos sobre dr y dz,

$$E_z = \frac{1}{4\pi\epsilon_0} \rho_0 (2\pi) \int_0^R dr \int_{-H/2}^{+H/2} \frac{r(z - z')}{[r^2 + (z - z')^2]^{3/2}} \, dz'$$

$$E_z = \frac{1}{4\pi\epsilon_0} \rho_0 (2\pi) \left[\int_{-H/2}^{+H/2} (z - z')\, dz\right] \times \left[\frac{1}{2} \int_0^R \frac{2r}{[r^2 + (z - z')^2]^{3/2}} \, dr\right]$$

$$E_z = \frac{1}{4\pi\epsilon_0} \rho_0 (2\pi) \left[ \int_{-H/2}^{+H/2} (z - z')\, dz' \right] \times \frac{1}{2} \left[ \frac{[r^2 + (z-z')^2]^{-3/2+1}}{\left(-\frac{3}{2}+1\right)} \right]_0^{R}$$

$$E_z = \frac{1}{4\pi\epsilon_0} \rho_0 (2\pi) \left[ \int_{-H/2}^{+H/2} (z - z')\, dz' \right] \times$$

$$\times \frac{1}{2} \left[ \left(-\frac{1}{2}\right)^{-1} \left( \frac{1}{[R^2 + (z-z')^2]^{1/2}} - \frac{1}{(z - z')} \right) \right]$$

$$E_z = \frac{1}{4\pi\epsilon_0} \rho_0 (2\pi) (-1) \int_{-H/2}^{+H/2} (z-z') \left[ \frac{1}{[R^2 + (z-z')^2]^{1/2}} - \frac{1}{(z-z')} \right] dz'$$

$$E_z = \frac{\rho_0 (2\pi)}{4\pi\epsilon_0} (-1) \left[ \int_{-H/2}^{+H/2} \frac{(z-z')}{[R^2 + (z-z')^2]^{1/2}} dz' - \int_{-H/2}^{+H/2} dz' \right]$$

cambio de variable

$$E_z = \frac{\rho_0 (2\pi)}{4\pi\epsilon_0} \left[ H - \int_{-H/2}^{+H/2} \frac{(z-z')}{[R^2 + (z-z')^2]^{1/2}} dz' \right] \qquad \begin{aligned} z - z' &= s \\ -dz' &= ds \end{aligned}$$

$$E_z = \frac{1}{4\pi\epsilon_0} \rho_0 (2\pi) \left[ H + \int_{s_{min}}^{s_{max}} \frac{s}{[R^2 + s^2]^{1/2}} ds \right] \qquad z' = H/2$$

$$E_z = \frac{1}{4\pi\epsilon_0} \rho_0 (2\pi) \left[ H + \frac{(R^2 + s^2)^{-1/2+1}}{(-1/2+1)} \right]_{s_{min} = \left(z + \frac{H}{2}\right)}^{s_{max} = \left(z - \frac{H}{2}\right)}$$

$z' = -H/2$

y con esto queda finalmente,

$$E_z = \frac{1}{4\pi\varepsilon_0}\,\rho_e\,(2\pi)\left[ H + \sqrt{R^2 + (z - H/2)^2} - \sqrt{R^2 + (z - H/2)^2} \right]$$