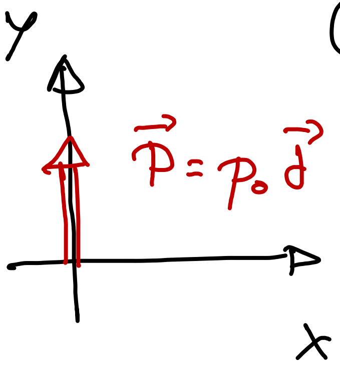

(a) Partimos de la expresión del potencial de un dipolo

Prob: 5.1

$$\phi = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3} = \frac{1}{4\pi\epsilon_0}\frac{p_0 y}{(x^2+y^2+z^2)^{3/2}}$$

y calculamos el campo eléctrico $\bar{E} = -\nabla\phi$ y las componentes x y z del gradiente son semejantes.

$$\nabla\phi = \frac{\partial\phi}{\partial x}\vec{i} + \frac{\partial\phi}{\partial y}\vec{j} + \frac{\partial\phi}{\partial z}\vec{k}$$

Evaluamos las derivadas:

$$\frac{\partial}{\partial x}\left[\frac{1}{r^3}\right] = \frac{\partial}{\partial r}(r^{-3})\frac{\partial r}{\partial x} = (-3)\,r^{-4}\frac{\partial}{\partial x}(x^2+z^2+y^2)^{1/2}$$

$$= -\frac{3}{r^4}\,\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-1/2}(\cancel{2}x) = -\frac{3x}{r^5}$$

y análogamente $\qquad \dfrac{\partial}{\partial z}\left[\dfrac{1}{r^3}\right] = -\dfrac{3z}{r^5}$

Para la componente y del gradiente:

$$\frac{\partial}{\partial y}\left[\frac{y}{r^3}\right] = \frac{1}{r^3} + y\frac{\partial}{\partial y}(r^{-3}) = \frac{1}{r^3} + y\left(-\frac{3y}{r^5}\right)$$

Si sumamos ahora todas las componentes queda

$$\vec{E} = -\nabla\phi = \frac{1}{4\pi\epsilon_0}\left[-\frac{3x}{r^5}\vec{i} + \left(\frac{1}{r^3} - \frac{3y^2}{r^5}\right)\vec{j} - \frac{3z}{r^5}\vec{k}\right]$$

$$\vec{E} = \frac{(-p_0)}{4\pi\epsilon_0}\left[(-3)\,\frac{x\vec{i} + y^2\vec{j} + zk}{r^5} + \frac{1}{r^3}\vec{j}\right]$$

para $\vec{r} = d\vec{j}$ se tiene $r = d$ y queda

$$\vec{E} = \frac{(-p_0)}{4\pi\epsilon_0}\left[(-3)\,\frac{d^2}{d^5}\vec{j} + \frac{1}{d^3}\vec{j}\right] = \frac{(-p_0)}{4\pi\epsilon_0}\left[-\frac{3}{d^3} + \frac{1}{d^3}\right]\vec{j}$$

$$\vec{E} = \frac{(-p_0)}{4\pi\epsilon_0}\left(-\frac{2}{d^3}\right)\vec{j} = \frac{2p_0}{4\pi\epsilon_0 d^3}\vec{j} \qquad \vec{E} = \frac{p_0}{2\pi\epsilon_0 d^3}\vec{j}$$

y la fuerza sobre la carga q es $\vec{F} = q\vec{E}$

(b) El trabajo será $W = (-q)\Delta\phi = (-q)[\phi_a - \phi_b]$

$\vec{r}_a = d\vec{j}$ y $\vec{r}_b = 2d\vec{j}$ y empleando el potencial del dipolo tendremos

$$\phi_a = \phi(\vec{r}_a) = \frac{1}{4\pi\epsilon_0}\left[\frac{\vec{p}\cdot\vec{r}_a}{r_a^3}\right] = \frac{1}{4\pi\epsilon_0}\frac{p_0 d}{d^3} = \frac{p_0}{4\pi\epsilon_0 d^2}$$

$$\phi_b = \phi(\bar{r}_b) = \frac{1}{4\pi\epsilon_0}\left[\frac{\vec{p}\cdot\bar{r}_b}{r_b^3}\right] = \frac{1}{4\pi\epsilon_0}\ \frac{p_0(2d)}{(2d)^3} = \frac{p_0}{16\pi\epsilon_0 d^2}$$

$$W = (-q)\left[\frac{p_0}{4\pi\epsilon_0 d^2} - \frac{p_0}{16\pi\epsilon_0 d^2}\right] = \frac{-q p_0}{4\pi\epsilon_0 d^2}\left[1 - \frac{1}{4}\right] = \frac{3 q p_0}{16\pi\epsilon_0 d^2}$$