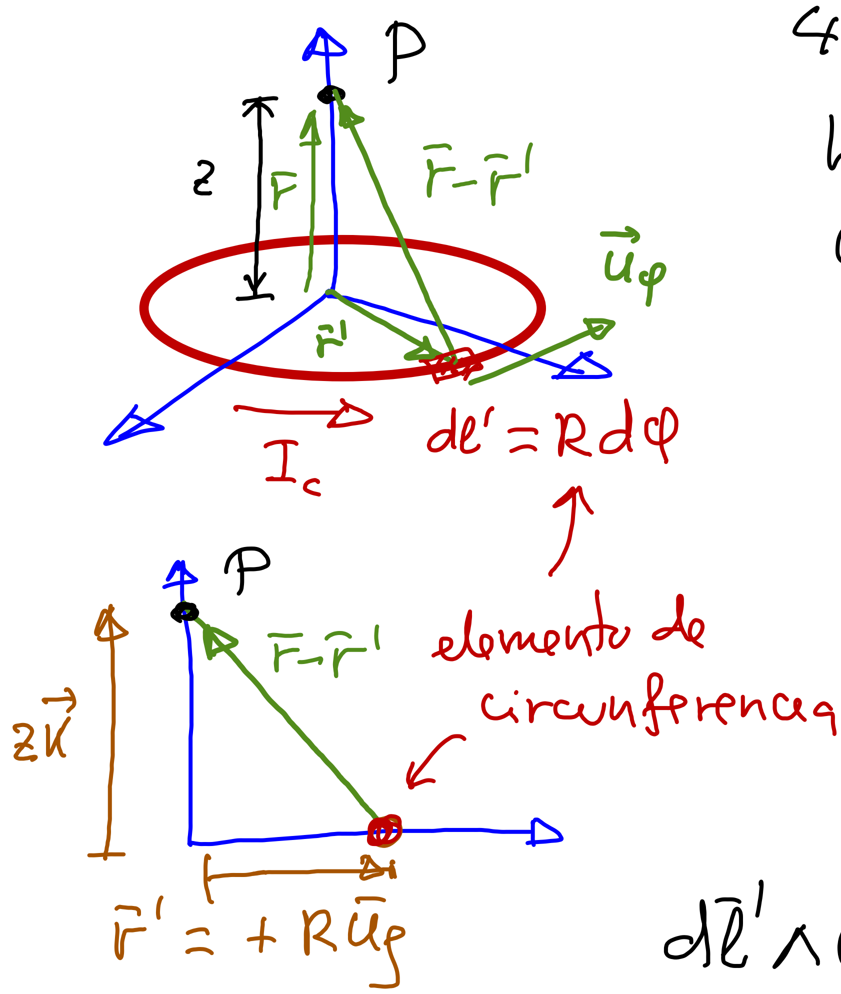
Calculamos el campo empleando la ley de Biot-Savart

Prob: 7.4

$$\vec{B} = \frac{\mu_0 I_c}{4\pi} \oint \frac{d\vec{\ell}' \wedge (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}$$



Los vectores involucrados son en este caso (ver dibujo) para ptos del eje Z

$$\begin{cases} d\vec{\ell}\,' = d\ell'\,\vec{u}_\varphi = (R\,d\varphi)\,\vec{u}_\varphi \\ \vec{r} - \vec{r}\,' = z\vec{k} - R\vec{u}_\rho \qquad \vec{r}\,' = R\vec{u}_\rho \\ |\vec{r} - \vec{r}\,'|^3 = (z^2 + R^2)^{3/2} \end{cases}$$

Tenemos entonces

$$d\vec{\ell}\,' \wedge (\vec{r} - \vec{r}\,') = (d\ell'\,\vec{u}_\varphi) \wedge (z\vec{k} - R\vec{u}_\rho)$$

$$d\vec{\ell}\,' \wedge (\vec{r} - \vec{r}\,') = z\,d\ell'\,\underbrace{(\vec{u}_\varphi \wedge \vec{k})}_{\vec{u}_\rho} - R\,d\ell'\,\underbrace{(\vec{u}_\varphi \wedge \vec{u}_\rho)}_{-\vec{k}}$$

Hemos de evaluar la integral:

$$\vec{B} = \frac{\mu_0 I_c}{4\pi} \oint \frac{(zR\,d\varphi)\,\vec{u}_\rho + (R^2\,d\varphi)\,\vec{k}}{[R^2 + z^2]^{3/2}}$$

$$\vec{B} = \frac{\mu_0 I_c}{4\pi} \left[\frac{Rz}{[R^2 + z^2]^{3/2}} \left(\oint \vec{u}_\rho\,d\varphi \right) + \frac{R^2\vec{k}}{[R^2 + z^2]^{3/2}} \underbrace{\left(\oint d\varphi \right)}_{= 2\pi} \right]$$

La primera integral es nula puesto que:

$$\oint \vec{u}_\rho \, d\varphi = \Big(\underbrace{\int_0^{2\pi} \cos\varphi \, d\varphi}_{=0} \Big) \vec{\imath} + \Big(\underbrace{\int_0^{2\pi} \operatorname{sen}\varphi \, d\varphi}_{=0} \Big) \vec{\jmath} = 0$$

y queda entonces,

$$\vec{B} = \frac{\mu_0 I_c}{4 \cancel{\pi}} \frac{R^2}{[R^2 + z^2]^{3/2}} \times (2\cancel{\pi}) \vec{k}$$

$$\vec{B} = \frac{R^2 \mu_0 I_c}{2 [R^2 + z^2]^{3/2}} \vec{k}$$

(b) En el centro del circulo $z = 0$ y se tiene

$$\vec{B} = \frac{R^2 \mu_0 I_c}{2 R^3} \vec{k} = \frac{\mu_0 I_c}{2R} \vec{k}$$