)

a) $\nabla P = \frac{\partial P}{\partial x}\vec{i} + \frac{\partial P}{\partial y}\vec{j} + \frac{\partial P}{\partial z}\vec{k} = \left(\frac{\partial P}{\partial r}\frac{\partial r}{\partial x}\right)\vec{i} + \left(\frac{\partial P}{\partial r}\frac{\partial r}{\partial y}\right)\vec{j} + \left(\frac{\partial P}{\partial r}\frac{\partial r}{\partial z}\right)\vec{k} =$

$= \left(f'\frac{x}{r}\right)\vec{i} + \left(f'\frac{y}{r}\right)\vec{j} + \left(f'\frac{z}{r}\right)\vec{k} = \frac{f'}{r}\left(x\vec{i} + y\vec{j} + z\vec{k}\right) = f'\frac{\vec{r}}{r}$

b) $\nabla \cdot \vec{w} = \frac{\partial w_x}{\partial x} + \frac{\partial w_y}{\partial y} + \frac{\partial w_z}{\partial z} = \frac{\partial}{\partial x}(xf) + \frac{\partial}{\partial y}(yf) + \frac{\partial}{\partial z}(zf) =$

$= \left(f + \frac{x^2}{r}f'\right) + \left(f + \frac{y^2}{r}f'\right) + \left(f + \frac{z^2}{r}f'\right) = 3f + rf'$

c)

$$\nabla \wedge \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xf & yf & zf \end{vmatrix} = \left[\frac{\partial(zf)}{\partial y} - \frac{\partial(yf)}{\partial z}\right]\vec{i} + \left[\frac{\partial(xf)}{\partial z} - \frac{\partial(zf)}{\partial x}\right]\vec{j} + \left[\frac{\partial(yf)}{\partial x} - \frac{\partial(xf)}{\partial y}\right]\vec{k}$$

$= \left(zf'\frac{y}{r} - yf'\frac{z}{r}\right)\vec{i} + \left(xf'\frac{z}{r} - zf'\frac{x}{r}\right)\vec{j} + \left(yf'\frac{x}{r} - xf'\frac{y}{r}\right)\vec{k} = 0$

d) $\nabla(\nabla \cdot \vec{w}) = \nabla(3f + rf') = \left[\frac{d}{dr}(3f + rf')\right]\frac{\vec{r}}{r} = (4f' + rf'')\frac{\vec{r}}{r}$

e) $\nabla^2 P = \nabla \cdot (\nabla P) = \nabla \cdot \left(f'\frac{\vec{r}}{r}\right) = 3\frac{f'}{r} + r\frac{d}{dr}\left(\frac{f'}{r}\right) =$

$= 3\frac{f'}{r} + r\left(\frac{f''}{r} - \frac{f'}{r^2}\right) = 2\frac{f'}{r} + f''$

f) $\nabla^2 \vec{w} = \nabla(\nabla \cdot \vec{w}) - \nabla \wedge (\nabla \wedge \vec{w}) = \nabla(\nabla \cdot \vec{w}) = (4f' + rf'')\frac{\vec{r}}{r}$