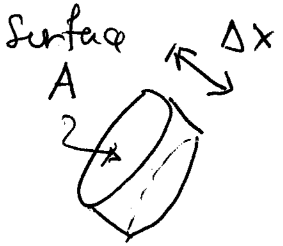
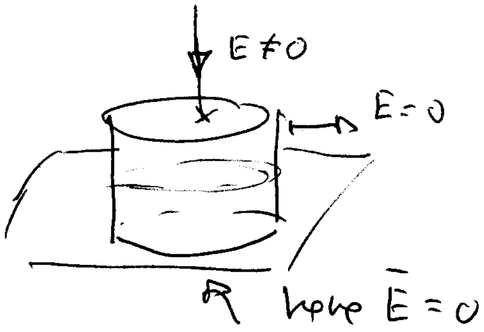
Plasma Frequency



$\left\{ \begin{array}{l} \text{Electrons inside the pillbox} \quad N_e = n_e \Delta V \\ \text{Electric charge within the pillbox: } Q = -e\, n_e\, \Delta V \end{array} \right.$ Volume: $\Delta V = A\, \Delta x$



$Q = -e\, n_e\, (A\, \Delta x)$ Aply Gauss' theorem:

$$\int_{\text{pillbox}} \bar{E} \cdot d\bar{s} = |\bar{E}| \times (A) = \int_{\substack{\text{vol.}\\ \text{pillbox}}} \frac{\rho}{\epsilon_0}\, dV = \frac{Q}{\epsilon_0}$$

$$\cancel{A}\, |E_x| = -e\, n_e\, \overbrace{\Delta V}^{\cancel{A}\Delta x} / \epsilon_0$$

$$\cancel{A} E = -\frac{e}{\epsilon_0} n_e (\cancel{A}\, \Delta x) \qquad E = -\frac{e\, n_e}{\epsilon_0}\, \Delta x \qquad \overbrace{\Delta x}^{\text{displacement}} = s$$

Equation of motion for electrons: $m_e \frac{d\vec{v}_e}{dt} = -e E_x$ $\quad m_e \frac{d^2 v_e}{dt} = -\frac{e^2 n_e}{\epsilon_0} s$

$$m \frac{d^2 s}{dt^2} + \frac{e^2 n_e}{\epsilon_0} s = 0 \qquad \ddot{s} + \left(\frac{e^2 n_e}{\epsilon_0 m_e} \right) s = 0 \qquad \omega_{pe} = \left(\frac{e^2 n_e}{\epsilon_0 m_e} \right)^{1/2}$$

electron plasma (langmuir) frequency.

Similar arguments apply to ions:

$$\omega_{pe} = \left(\frac{e^2 n_e}{\epsilon_0 m_e} \right)^{1/2} \qquad \omega_{pi} = \left(\frac{e^2 n_i}{\epsilon_0 m_i} \right)^{1/2} \qquad \frac{\omega_{pe}}{\omega_{pi}} = \sqrt{\frac{m_i}{m_e}}$$

Also $f_{pe} = \frac{\omega_{pe}}{2\pi}$ $f_{pi} = \frac{\omega_{pi}}{2\pi}$ $\frac{f_{pe}}{f_{pi}} = \sqrt{\frac{m_i}{m_e}}$ (2)

Electrons and ions times: $\begin{cases} \tau_{pe} \sim 1/f_{pe} & \text{(fast response)} \\ \tau_{pi} \sim 1/f_{pi} & \text{(slow response)} \end{cases}$

$\tau_{pe} << \tau_{pi}$

Multiple time scales in the plasma: the motion of ions is "frozen" on the time scale $\tau_i < \tau < \tau_e$

Plasma Parameter

$$E(r, v_\alpha) = \frac{m_\alpha v_\alpha^2}{2} - \frac{e^2}{4\pi\epsilon_0 r}$$ repelling particles

closest approach $E = 0 \Rightarrow r_c \rightarrow \frac{mv^2}{2} - \frac{e^2}{4\pi\epsilon_0 r} = 0 \rightarrow \frac{mv^2}{2} = \frac{e^2}{4\pi\epsilon_0 r_c}$

$$r_c = \frac{2e^2}{4\pi\epsilon_0 mv^2}$$ $V = \left(\frac{2kT}{m}\right)^{1/2}$ thermal speed.

$$r_c = \frac{2e^2}{4\pi\epsilon_0 \cancel{m}\left(\frac{kT}{\cancel{m}}\right)}$$ $$r_c = \frac{2e^2}{4\pi\epsilon_0 K_B T}$$ density $r_d = \frac{1}{n_d^{1/3}}$

(packing particles)

$$\langle E_{el} \rangle = \frac{e^2}{4\pi\epsilon_0 r_d} \qquad \Gamma = \frac{r_c}{r_d} = \frac{e^2 \sqrt[3]{n_0}}{4\pi\epsilon_0 K_B T}$$

$$\langle E_{th} \rangle \approx K_B T \qquad \Gamma = \langle E_{el} \rangle / \langle E_{th} \rangle$$

Large Γ → Strongly coupled plasma (3)

Low Γ → Weakly coupled plasma

The coupling parameter is related with other relevant quantities as:

$$\Gamma \approx \frac{e\phi}{K_B T} = \frac{e^2}{4\pi\epsilon_0 r_a} \times \frac{1}{K_B T} = \frac{1}{4\pi} \times \frac{e^2}{\epsilon_0 KT} \times n_0^{1/3} = \frac{1}{4\pi} \frac{n_0 e^2}{\epsilon_0 KT} \times \frac{n_0^{1/3}}{n_0}$$

Electrostatic energy of packed charges at distance $\Big\}\ e\phi(r_a)$ $\quad r_a \sim 1/\sqrt[3]{n_0}$

$$\Gamma \sim \frac{e\phi}{KT} = \frac{1}{4\pi} \frac{1}{\lambda_D^2} \times \frac{1}{n_0^{2/3}} = \frac{1}{4\pi} \frac{1}{(n_0 \lambda_D^3)^{2/3}}$$

$$\frac{1}{(n_0\lambda_D^3)^{2/3}} = \frac{1}{N_D^{2/3}}$$

Γ is related with the parameter $|e\phi/KT| \ll 1$ we used in the Debye Shielding. ⇒ small Γ