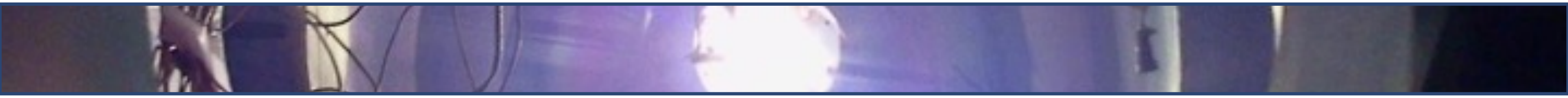

The ideal Maxwellian plasma

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Plasmas are, …

The plasma state of matter may be defined as a mixture of positively charged ions, electrons and neutral atoms which constitutes a *macroscopic electrically neutral medium* which *responds to the electric and magnetic fields in a collective mode*

Properties

- Charged particles interact through *long distance electromagnetic forces* in addition to short range molecular collisions.
- The density of negative n_e and positive n_i charged particles are equal, so that on the average *the medium is electrically neutral* (quasineutrality).
- The response to external perturbations is collective, large number of charges are involved.
- We may have *multicomponent plasmas*, ions with negative charge, dusty plasmas (complex plasmas), ...etc.

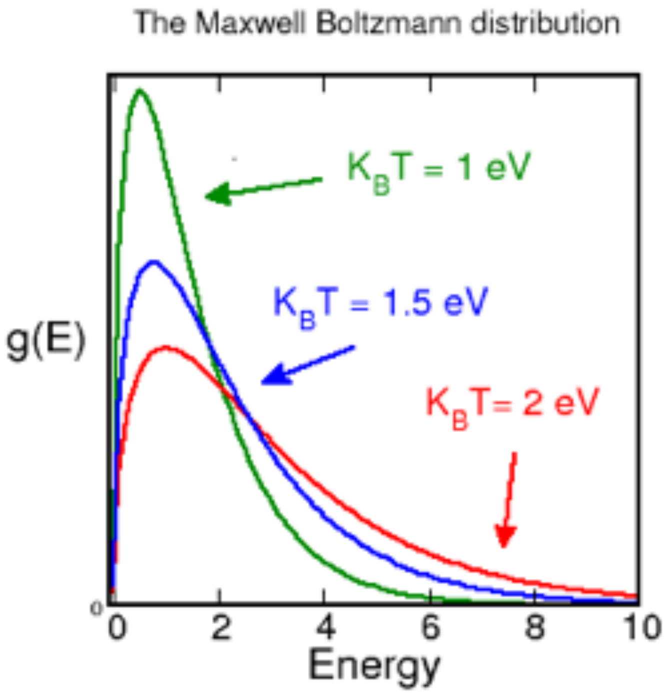
Physics is difficult, …

- For simplicity, we will limit to one component classical plasmas with single charged ions.
- The particles in our plasmas α= a, i, e will be electrons e , ions i and eventually neutral atoms a with number densities n_α
- The particle temperatures equivalent to the average kinetic energy of particles are usually expressed in eV and 1 eV = 11600 K,

$$\frac{e}{k_B T} = \frac{1.6 \times 10^{19}}{1.38 \times 10^{-23} \times 1} = 11,594 \simeq 11,600\,K$$

- The ion charge is Q_i = eZ but in most cases we will consider only single charged ions (Z=1).
- We will use MKSC unit system.

The particle energy distribution function of equilibrium system is Maxwellian with temperature $k_B T$



$$\begin{cases} f_a(\mathbf{v}) = \left(\dfrac{m_a}{2\pi\, k_B T}\right)^{3/2} \exp\left(-\dfrac{m_a \mathbf{v}^2}{2 k_B T}\right) \\ g_a(E) = \dfrac{2}{\sqrt{\pi}} \dfrac{\sqrt{E}}{(k_B T)^{3/2}} \exp\left(-E/k_B T\right) \end{cases}$$

The normalized probability distribution

$$\int_0^{\infty} g_a(E)\, dE = \int_{-\infty}^{+\infty} f_a(\mathbf{v})\, d\mathbf{v} = 1$$

$$\left.\begin{array}{l} dn_a = n_{ao}\, f_a(\mathbf{v})\, d\mathbf{v} \\ dn_a = n_{ao}\, g_a(E)\, dE \end{array}\right\} \; dn_a \quad \text{particles with} \; \begin{cases} (\mathbf{v},\ \mathbf{v} + d\mathbf{v}) \\ (E,\ E + dE) \end{cases}$$

Macroscopic properties, …

The macroscopic physical properties are calculated as averages of the particle energy distribution function. The average energy per particle becomes,

$$\langle E \rangle = \int_0^\infty E\, g_a(E)\, dE = \frac{3}{2}\, k_B T \qquad E_i = n_{ao}\, \langle E \rangle = \frac{3}{2}\, n_{ao}\, k_B T$$

The thermal speed redefines the velocity distribution function,

$$v_{th} = \sqrt{\frac{2\, k_B T}{m_a}} \qquad f_a(\mathbf{v}) = \frac{1}{(\sqrt{\pi}\, v_{th})^3}\, \exp\left(-\mathbf{v}^2/v_{th}^2\right)$$

The velocity averages,

$$\langle v_x \rangle = \langle v_y \rangle = \langle v_z \rangle = 0$$

$$v_{ave} = \langle \sqrt{\mathbf{v} \cdot \mathbf{v}}\, \rangle = \sqrt{\frac{8 k_B T}{\pi\, m_a}} \qquad \langle |v_x| \rangle = \sqrt{\frac{2\, k_B T}{\pi\, m_a}}$$

define the particle flux,

$$\Gamma_a = \frac{1}{2}\, n_{ao}\, v_{ave} \qquad J_\alpha = q_\alpha\, \Gamma_\alpha = q_\alpha\, \frac{1}{2}\, n_{ao}\, v_{ave}$$

The equilibrium state of ideal Maxwelian plasmas, …

- The energy distribution function of charged particles ions and electrons is Maxwellian.

- All particle groups have the same kinetic temperature $K_B T_e = K_B T_i = K_B T$ and $K_B T$ is the temperature corresponding to the thermodynamic equilibrium.

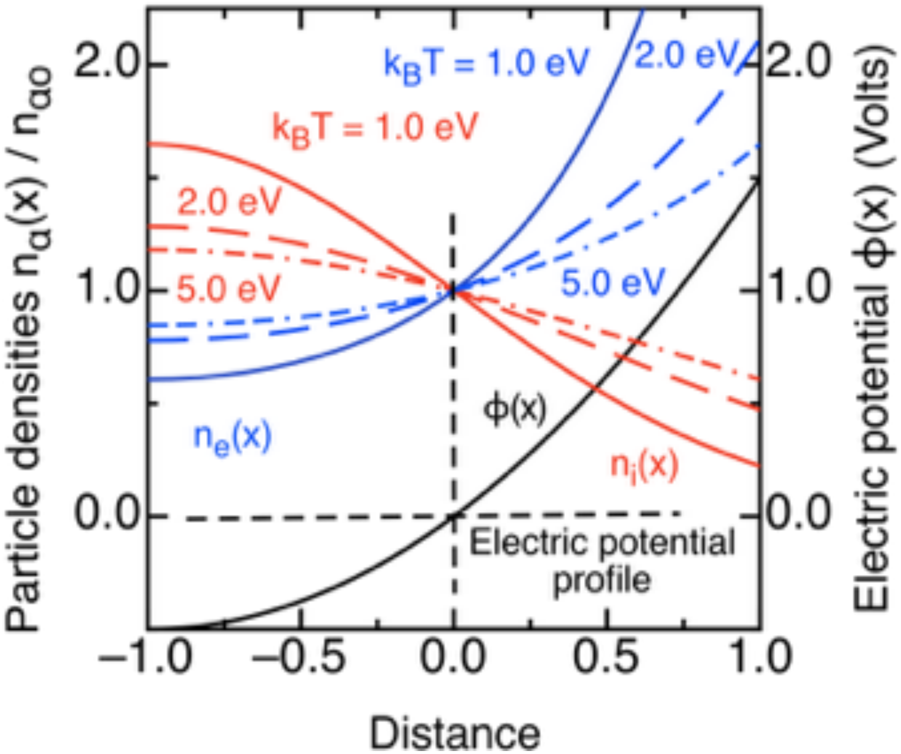
- Plasma quasineutrality, $n_e \approx n_i$ implies that no electric fields exists in the equilibrium plasma bulk; uniform plasma potential.

$$\nabla \cdot \boldsymbol{E} = \frac{e}{\epsilon_o} (n_i - n_e) \simeq 0 \qquad \boldsymbol{E} \simeq 0$$

- No transport; no particle currents.

The Maxwellian plasma in an external electric field, …

Background equilibrium,



$$E_\alpha = \frac{m_\alpha \mathbf{v}^2}{2} + q_\alpha \phi(x)$$

$$n_\alpha = n_{\alpha o} \int_0^\infty g_\alpha(E)\, dE$$

$$\begin{cases} n_e(x) = n_{eo} \exp\left(\dfrac{e\phi}{k_B T}\right) \\ n_i(x) = n_{io} \exp\left(-\dfrac{e\phi}{k_B T}\right) \end{cases}$$

In *cold plasma* $k_BT \approx 0$ the charge separation is complete while in a *finite temperature plasma* $k_BT \neq 0$ the thermal and electrostatic energies compete. This permits *to shield out* the small amplitude fluctuations of electric and magnetic fields.