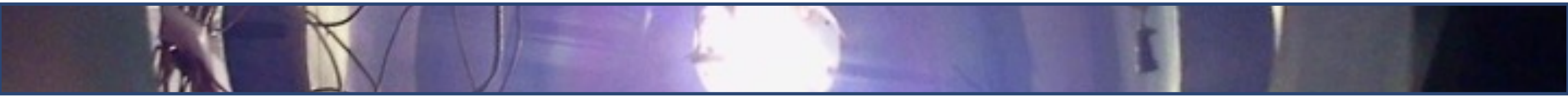

The plasma length and time scales

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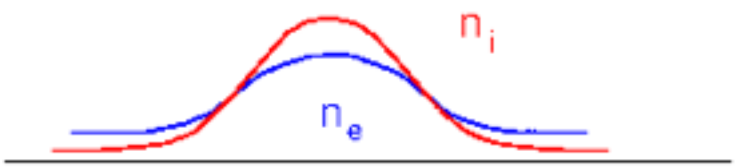
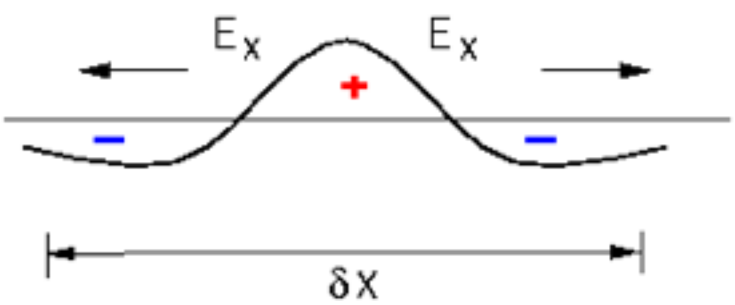
Plasma length and time scales, …

The damping and/or attenuation of small amplitude fluctuations of the equilibrium takes place over characteristic length and time scales.

$$\nabla \cdot \boldsymbol{E} = \frac{e}{\epsilon_o}\,(n_i - n_e) \qquad n_i \neq n_e \qquad \boldsymbol{E} \simeq \boldsymbol{E}_1$$ small amplitude perturbations



- Space fluctuations: Debye length
- Time fluctuations: Plasma frequency
- Number of charges: Plasma parameter



$$\lambda_{De} = \sqrt{\frac{\epsilon_o\, k_B T}{e^2\, n_{eo}}} \qquad \lambda_{Di} = \sqrt{\frac{\epsilon_o\, k_B T}{e^2\, n_{io}}}$$

$$k_B T_e \gg k_B T_i \qquad \lambda_{De} \gg \lambda_{Di}$$

$$\omega_{pi} = \sqrt{\frac{e^2\, n_{io}}{\epsilon_o\, m_i}} \qquad \omega_{pe} = \sqrt{\frac{e^2\, n_{eo}}{\epsilon_o\, m_e}} \qquad \frac{\omega_{pi}}{\omega_{pe}} = \sqrt{\frac{m_e}{m_i}}$$

The Debye length and shielding, …

The initial equilibrium: $\mathbf{E}_o = 0$ $\quad n_{eo} \simeq n_{io} = n_o$ $\quad k_B T_e; k_B T_i$

where we introduce an small perturbation in the electric charge,

$$\left.\begin{aligned} \delta\rho_{ext} &= q\,\delta(\mathbf{r}) \\ \delta\rho_{sp} &= e\left[n_i(\mathbf{r}) - n_e(\mathbf{r})\right] \end{aligned}\right\}$$

$\mathbf{E}(\mathbf{r}) \simeq \mathbf{E}_o + \mathbf{E}_1(\mathbf{r}) \quad \mathbf{E}_1(\mathbf{r}) = -\nabla\varphi_1(\mathbf{r})$

and ***E**$_l$(**r**)* represents the *perturbed electric field*.

The field ***E**$_l$(**r**)* is governed by the Poisson equation, $\nabla \cdot \mathbf{E} = \dfrac{\delta\rho_{ext} + \delta\rho_{sp}}{\epsilon_o}$

$$\nabla \cdot \mathbf{E}_1 = \frac{q}{\epsilon_o}\delta(\mathbf{r}) + \frac{e}{\epsilon_o}[n_i(\mathbf{r}) - n_e(\mathbf{r})])$$

where, $n_\alpha(\mathbf{r}) = n_{\alpha o}\exp\left(\pm\dfrac{e\,\varphi_1(\mathbf{r})}{k_B T_\alpha}\right)$

Small amplitude perturbations of the charge/electric field means that the *thermal energy |e φ(**r**)|* dominates over the *electrostatic energy* *$k_B T$*

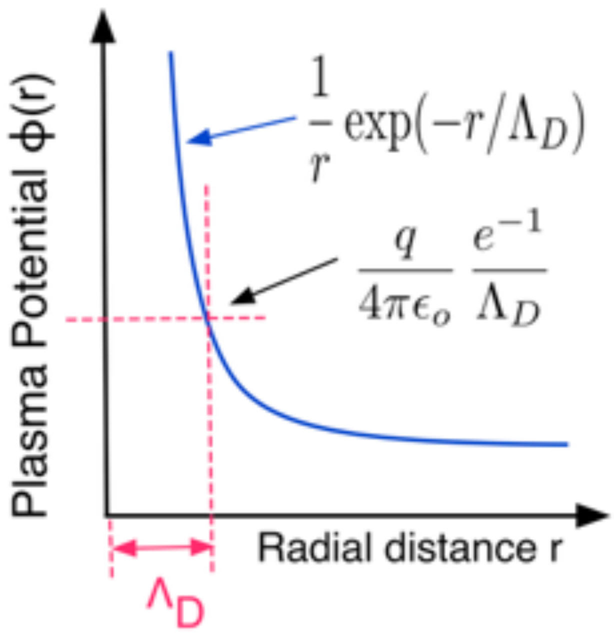
$\alpha = e, i$ $\quad \left|\dfrac{e\,\varphi_1(\mathbf{r})}{k_B T_\alpha}\right| \ll 1$ $\quad$ we approximate $\quad n_\alpha(\mathbf{r}) \simeq n_{\alpha o}\left(1 \pm \dfrac{e\,\varphi_1(\mathbf{r})}{k_B T_\alpha}\right)$

Debye shielding, ...

$$-\nabla^2\varphi_1(\mathbf{r}) = \frac{1}{\epsilon_o}\left[\delta\rho_{ext} + \left(\frac{e^2\,n_o}{k_B T_e}\,\varphi_1(\mathbf{r})\right) + \left(\frac{e^2\,n_o}{k_B T_i}\,\varphi_1(\mathbf{r})\right)\right] \qquad \alpha = e, i$$

that introduces the Debye lengths, $\lambda_{D\alpha} = \sqrt{\frac{\epsilon_o k_B T_\alpha}{e^2\,n_o}}$ Setting,

$$\frac{1}{\Lambda^2} = \frac{1}{\lambda_{De}^2} + \frac{1}{\lambda_{Di}^2} \qquad \left(\nabla^2 - \frac{1}{\Lambda^2}\right)\varphi_1(\mathbf{r}) = -\frac{q}{\epsilon_o}\delta(\mathbf{r}) \qquad \varphi_1(\mathbf{r}) = \frac{q}{4\pi\epsilon_o}\frac{e^{-r/\Lambda}}{r}$$



The perturbations of the electric charge and/or plasma potential decay in space with an exponential rate governed by λ_D

The Debye length accounts for thermal effects, the size ~λ_D of the perturbed region increases with $k_B T$

This is a linearization only valid whereas,

$$\left|\frac{e\,\varphi_1(\mathbf{r})}{k_B T_\alpha}\right| \ll 1$$

We deal with a non linear Poisson equation otherwise,

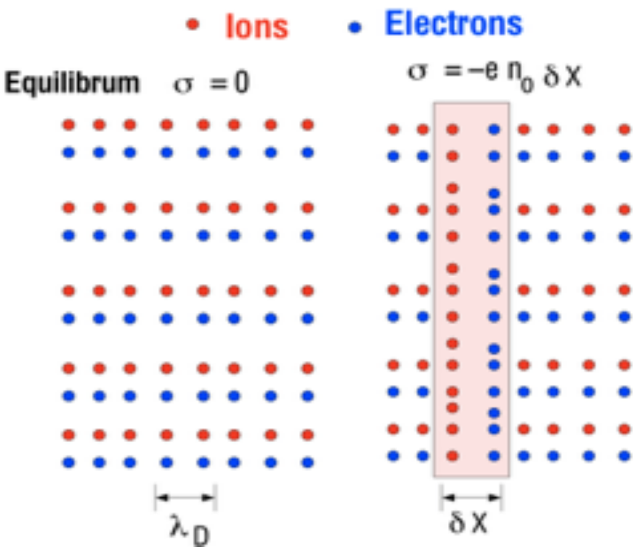
Approximate magnitudes in typical plasmas, …

Plasma	n_e (cm^{-3})	$K_B T_e$ (eV)	λ_{De} (cm)	f_{pe} (Hz)	$n_e \lambda_{De}^3$
Interstellar gas	1	1	700	$6{,}0 \times 10^4$	4×10^8
Solar corona	10^9	100	0,2	$2{,}0 \times 10^9$	8×10^6
Solar atmosphere *Gas discharge*	10^{14}	1	$7{,}0 \times 10^{-5}$	$6{,}0 \times 10^{11}$	40
Tokamak	10^{14}	10^4	$2{,}0 \times 10^{-3}$	$2{,}0 \times 10^{12}$	6×10^6

Values from *NRL Plasma Formulary 2007*. This practical reference could be downloaded for free at; http://wwwppd.nrl.navy.mil/nrlformulary/

Time scale: the plasma frequency, …

Again the initial equilibrium: $\mathbf{E}_o = 0 \quad n_{eo} \simeq n_{io} = n_o \quad k_B T_e ; k_B T_i$



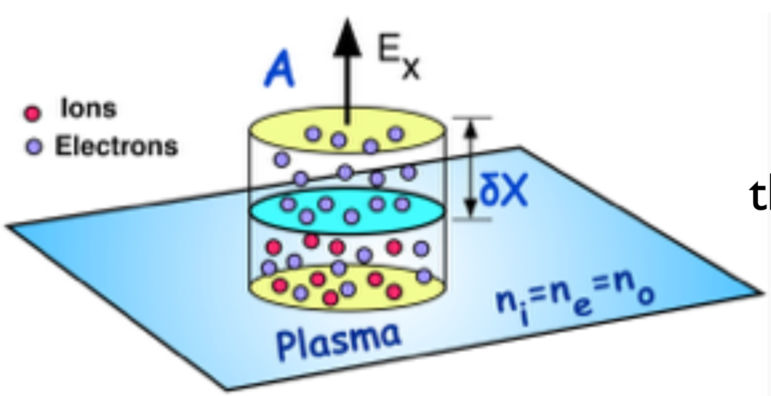
and we consider a one small perturbation of the electric charge in one dimension as in the figures,

$$\rho = -e\, n_o (A\, \delta X)$$

$$\int_S \mathbf{E}_1 \cdot d\mathbf{s} = A\, E_{1x} = \frac{\rho}{\epsilon_o} = -\frac{e}{\epsilon_o} n_o (A\, \delta X)$$

The electric field, $E_{1x} = -\frac{e}{\epsilon_o} n_o\, \delta X$

gives us an equation of motion for ions or electrons



$$\frac{d^2}{dt^2}(\delta X) + \left[\frac{e^2 n_o}{m_\alpha \epsilon_o}\right] (\delta X) = 0$$

that defines the electron/ion plasma frequencies,

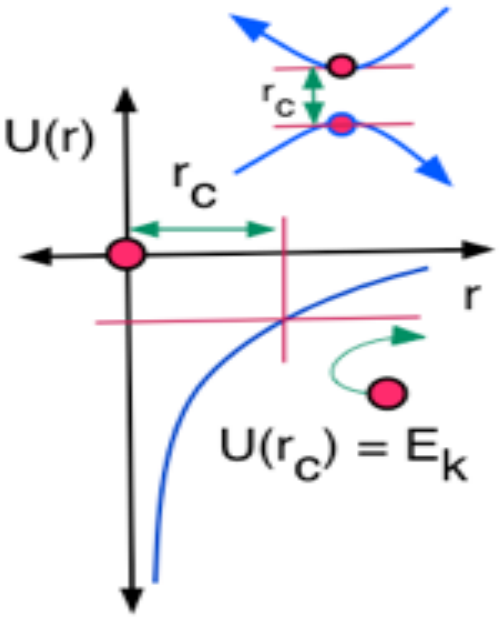
$$\alpha = e, i \qquad \omega_{p\alpha} = \sqrt{\frac{e^2\, n_o}{m_\alpha\, \epsilon_o}}$$

The plasma and coupling parameters, …

In order to shield out the electric field perturbations a minimum number of charges are required inside a sphere of radius λ_D. This defines the plasma parameter N_D for ions and electrons,

$$N_{De} = \frac{4\pi}{3} n_o \lambda_{De}^3 \qquad k_B T_e \gg k_B T_i \qquad \lambda_{De} \gg \lambda_{Di} \qquad N_{De} \gg N_{Di}$$

The plasma parameter is closely related with *the coupling parameter* Γ_C the ratio between thermal and electrostatic energies.



$$\Gamma_C = \frac{r_c}{r_d}$$

- $r_d \sim n_o^{-1/3}$ average packing of charges
- r_c is the closest colliding approach as in the figure

$$E(\mathbf{r}, \mathbf{v}_\alpha) = \frac{m_\alpha v_\alpha^2}{2} - \frac{e^2}{4\pi\epsilon_o r} \qquad E(r_c, v_{th}) = 0$$

$$r_c = \frac{e^2}{4\pi\epsilon_o k_B T} \qquad \Gamma_C = \frac{e^2 n_o^{1/3}}{4\pi\,\epsilon_o\,k_B T}$$

Coupling parameter, …

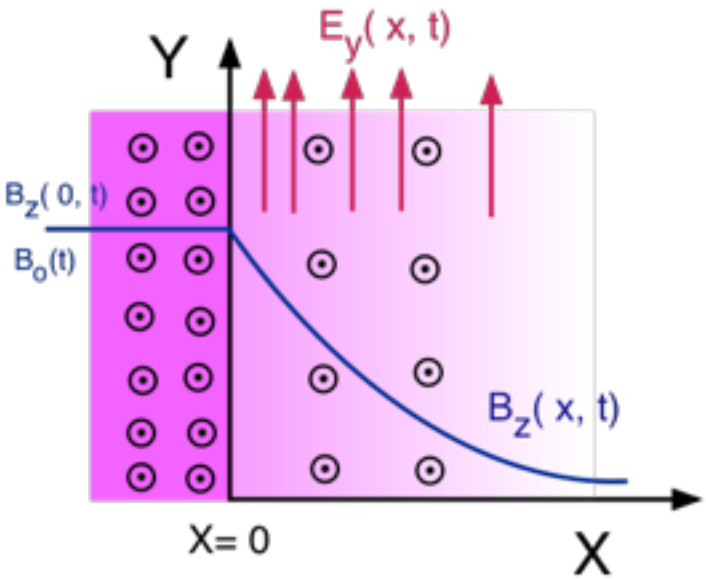
For the coupling parameter we also have,

$$\Gamma_c^3 = \frac{1}{(4\pi)^3} \times \frac{1}{n_o^2} \times \left(\frac{n_o\, e^2}{\epsilon_o\, k_B T} \right)^3 = \frac{1}{(4\pi)^3} \frac{1}{n_o^2\, \lambda_D^6} = \frac{1}{(4\pi \times 9)} \left(\frac{3}{4\pi\, n_o \lambda_D^3} \right)^2$$

and we obtain, $\Gamma_c = \dfrac{1}{36\,\pi\, N_D^2}$ equivalent to, $\Gamma_C = \dfrac{e^2\, n_o^{1/3}}{4\pi\, \epsilon_o\, k_B T}$

Description	*Plasma parameter*	
Limits	$\Gamma_C \gg 1$ ($N_D \ll 1$)	$\Gamma_C \gg 1$ ($N_D \gg 1$)
Coupling	Strongly coupled	Weakly coupled
Debye sphere	Sparsely populated	Densely populated
Electrostatic influence	Strong	Weak
Characteristic	Cold and dense	Hot and diffuse
Examples	Laser ablation plasmas Inertial fusion experiments White dwarfs Neutron stars	Ionospheric plasmas Magnetic fusion experiments Space plasmas Electric discharge plasmas

The plasma skin depth, …



We consider the plasma electrons

$$\mathbf{B} = B_z(x,t)\,\mathbf{u}_z \qquad B_z(0,t) = B_o(t)$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \wedge \mathbf{E} \qquad \mathbf{E}(x,t) = E_y(x,t)\,\mathbf{u}_y$$

We also have,

$$\nabla \wedge \mathbf{B} = \mu_o \mathbf{J_e} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} \simeq -e\,n_{eo}\,\mathbf{v}_e$$

and the equation of motion for electrons

$$\frac{dv_{ey}}{dt} = -\frac{e}{m_e}E_y(x,t)$$

$$\frac{\partial B_z}{\partial x} = e\,\mu_o\,n_{eo}\,\mathbf{v}_e$$

$$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x}$$

$$\frac{\partial^2 E_y}{\partial x^2} = \frac{e^2}{m_e} n_{eo}\mu_o\,E_y = \frac{1}{c^2}\left(\frac{e^2\,n_{eo}}{\epsilon_o\,m_e}\right)E_y$$

$$\frac{\partial^2 E_y}{\partial x^2} = \left(\frac{\omega_{pe}}{c}\right)^2 E_y \qquad B_z(x,t) = B_o(t)\,e^{-x/\lambda_p}$$

$$E_y(x,t) = \lambda_p\,(\partial B_o/\partial t)\,e^{-x/\lambda_p}$$

$$\lambda_p = \sqrt{\frac{c}{\omega_{pe}}}$$

The magnetic field exponentially decreases in the plasma along the so called *skin depth* of *London characteristic length*

Magnetized plasmas: Larmor radius, …

The force experienced by electric charges in this course (non-relativistic),

$$\alpha = e, i \qquad \mathbf{F}_\alpha = q_\alpha \, n_\alpha \, (\mathbf{E} + \mathbf{v}_\alpha \wedge \mathbf{B})$$

this gives the cyclotron frequency, $\Omega_\alpha = q_\alpha \, B_\perp \, / \, m_\alpha$ and using the perpendicular speed to the magnetic field lines we obtain the *Larmor radius*

$$r_{L,\alpha} = \frac{m_\alpha \, v_\perp}{|q_\alpha| \, B}$$

a new length is defined using the particle thermal speed

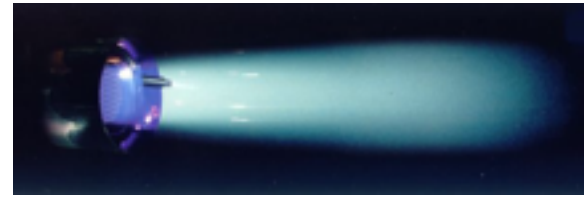
$$R_{L,\alpha} = \frac{V_{th,\alpha}}{\Omega_\alpha}$$

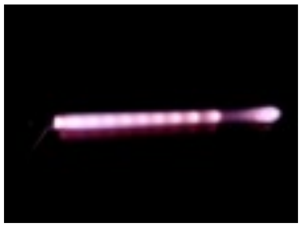
$$R_{L,i} = \sqrt{\frac{m_i}{m_e}} R_{L,e}$$

Magnetized electrons and unmagnetized ions $\quad R_{L,e} \ll R_{L,i} > L$

Magnetized electrons and ions $\quad R_{L,e} \ll R_{L,i} < L$

Orders of magnitude, …





Low pressure discharges

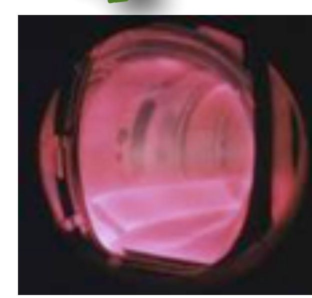
$KT \approx 1\text{-}3$ eV, $n \approx 10^8$ cm^{-3}

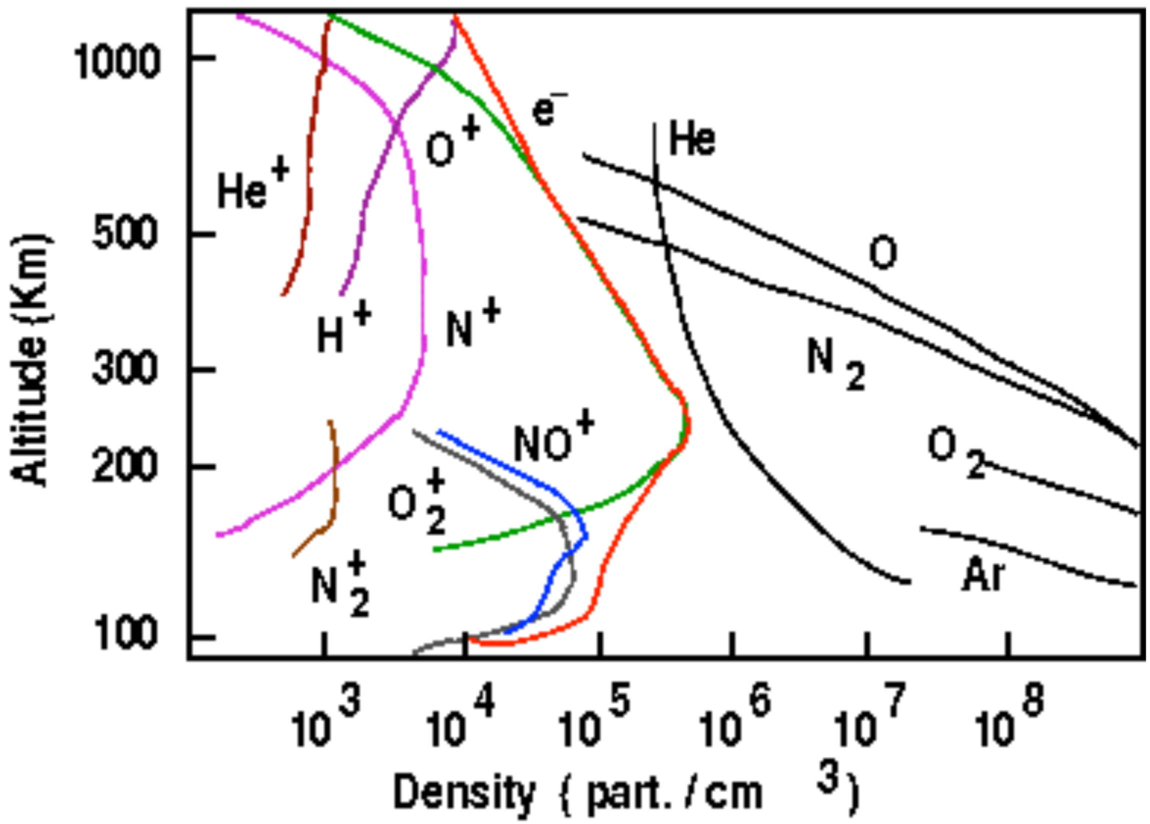
Solar corona

$KT \approx 100$ eV, $n \approx 10^9$ cm^{-3}

Fusion reactor

$KT \approx 10^4$ eV, $n \approx 10^{15}$ cm^{-3}





The black curves are for neutral particles and the red line is the altitude dependent electron density.

The sum of all different ion densities n_i equals the electron density n_e

The ions are produced by the absorption by the neutrals of parts of the solar radiation spectrum. The maximum rate takes place at the F_1 and F_2 peaks.