An Introduction to Plasma Physics and its Space Applications, Volume 1

Fundamentals and elementary processes

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Chapter 2

Ionized gases and plasmas

Plasmas are mixtures of ions, electrons and neutral atoms moving randomly where collisions at the atomic and molecular levels play a crucial role. The ions and electrons interact through *long-range* Coulomb forces, whereas neutral atoms and molecules interact by means of *short-range* collisions. In this chapter we introduce a definition of the plasma state of condensed matter, useful basic concepts and the characteristics of plasma particle interactions.

Collisions at the microscopic particle level mark the differences between partially ionized ($\alpha_g > 0$) and ordinary electrically neutral gases ($\alpha_g = 0$). In the physical systems of our interest, particle velocities $\boldsymbol{v}$ are large enough so that their associated quantum wavelengths $\lambda = h/mv$ are negligible and hence physical effects at the atomic scale can be disregarded. Since ions and electrons in plasmas of our interest have moderate energies, their collisions can be described using non-relativistic models. In addition, we can neglect the effect of Earth's gravity on the trajectories of atoms and molecules at the laboratory scale.

Under these conditions, atoms or molecules can be represented as spherically symmetrical force centers that perform a zig-zag motion, illustrated in figure 2.1(a), exchanging energy and momentum through collisions. When gas particles approach distances comparable to their size intense electric fields develop at the atomic level that change their original directions of motion. Between two successive encounters, their trajectories are straight lines abruptly altered by *short-range collisional interactions*, as shown in figure 2.1(a). At low gas pressures molecular encounters are essentially *binary* and collisions involving more than two particles are unlikely events that can be disregarded.

The most likely event for low molecular velocities is the *elastic collision* where the initial state of the particles after the encounter remains unaltered and both the energy and momentum are conserved. At higher speeds a fraction of the initial kinetic energy might be employed to produce internal changes of colliding particles. In these *inelastic collisions* the momentum is conserved but not the kinetic energy,

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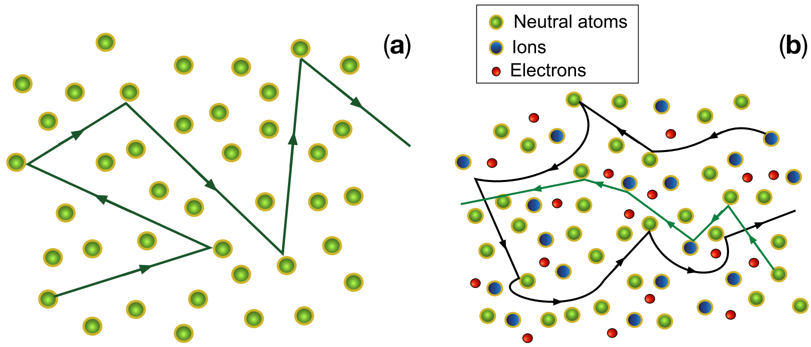

Figure 2.1. Schematic of the random motion of particles in a neutral gas where only *short-range* collisions are present and in a partially ionized gas where *short-range* and *long-range* Coulomb collisions coexist. (a) The random motion of a neutral atom into its parent gas where trajectories right paths. (b) The random motion of an ion and a neutral atom into a partially ionized gas.

that can produce for example the impact ionization of molecules, transitions between their electronic levels or other internal changes.

In addition to neutral atoms, *ionized gases* contain ions and electrons that create intense electromagnetic fields on a microscopic scale. The electric fields created by charged particles do not affect the trajectories of neutrals that still are again straight lines between two molecular encounters, as in figure 2.1(a). As shown by the scheme of figure 2.1(b), the motion of electrically neutral particles is not affected by the electromagnetic fields and is again dominated by *short-range* collisions, that now also involve electrons and ions.

In contrast, *long-range electromagnetic forces* involve a huge number of charged particles located at distant points and strictly speaking are not *binary collisions*. The motions of ions and electrons are strongly influenced by the average electric field at the microscopic scale produced by other charged particles. These electromagnetic forces among ions and electrons are *long-range interactions* (also called Coulomb collisions) that are mainly responsible for the momentum and energy exchange between charged species. This is the origin of a *collective response* of ionized gases to external and/or internal electromagnetic fields.

The *collective response* to electromagnetic fields involving a number of charged particles constitutes a distinctive feature of the plasma state of condensed matter. In partially ionized gases, both short and long range collisional interactions coexist and convey momentum and energy, coupling the motion of neutral and charged particles (box 1.1).

2.1 Collisions and elementary processes

Molecular collisions in plasmas involve physical processes such as light emission, ionization of neutral atoms, momentum and energy transfer between plasma species, etc. Furthermore, plasmas are chemically active media due to the presence of free

Box 2.1. Energy transfer in elastic collisions

In the frame where m_b remains at rest ($\boldsymbol{v}_b = 0$) the mass m_a moves at speed $\boldsymbol{v}_a$ and its trajectory is symmetric with respect to the line AB in figure 2.2, d_{ab} is the closest distance of approach. The speed of m_a is $\boldsymbol{v}_a'$ after the interaction pointing along the line that forms an angle θ_a and m_b moves with speed $\boldsymbol{v}_b'$.

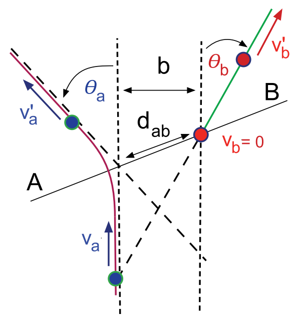

Figure 2.2. Elastic collision.

The kinetic energy and momentum are conserved magnitudes in the elastic collisions and along the dotted line in figure 2.2 (indicated by $\boldsymbol{v}_a$) and its perpendicular direction we have,

$$m_a\, v_a = m_a\, v_a' \cos\theta_a + m_b\, v_b' \cos\theta_b$$
$$0 = m_a\, v_a' \sin\theta_a - m_b\, v_b' \sin\theta_b$$

where $v_a > v_a'$ then,

$$v_a' \cos\theta_a = v_a - \frac{m_b}{m_a} v_b' \cos\theta_b \quad \text{and,} \quad v_a' \sin\theta_a = \frac{m_b}{m_a} v_b' \sin\theta_b$$

Both equations can be squared to eliminate the angle θ_a and we obtain,

$$v_a'^2 = v_a^2 + \left(\frac{m_b}{m_a}\right)^2 v_b'^2 - 2\left(\frac{m_b}{m_a}\right) v_b'^2\; v_a \cos\theta_b$$

This expression can be introduced in the kinetic energy conservation equation,

$$\frac{m_a\, v_a^2}{2} = \frac{m_a\, v_a'^2}{2} + \frac{m_b\, v_b'^2}{2}$$

to eliminate the speed v_a' and we obtain the following relation between v_b' and v_a,

$$v_b' = \frac{2\, m_a}{m_a + m_b} v_a \cos\theta_b \quad \text{then,} \quad E_b' = E_a \frac{4\, m_a\, m_b}{(m_a + m_b)^2} \cos^2\theta_b$$

where $E_b' = m_b\, v_b'^2/2$ is the kinetic energy transferred to m_b in the elastic collision in the frame where m_b was initially at rest. The maximum energy exchange for $\theta_b = (0, \pi)$ defines the ratio,

$$\delta_E = \frac{E_b'}{E_a} = \frac{4\, m_a\, m_b}{(m_a + m_b)^2} \tag{2.1}$$

ions and electrons that can produce reactions between atoms and molecules. We can define the *elementary plasma processes* as the molecular or atomic encounters accompanied by momentum, energy transfer and/or the transformation of colliding particles. In chapters 5 and 6 we discuss their physical description as well as atomic and molecular collisions of relevance in plasmas.

These elementary processes (or reactions) can be divided into *elastic* when the initial states of colliding particles remain unaltered by the encounter and *inelastic* otherwise. In the latter a part of the kinetic energy of colliding particles is transformed into changes of their initial states and/or internal energies. In the so-called *superelastic collisions* the internal energy of excited atoms or molecules can also be transformed into kinetic energy.

In the following we will call *photoprocesses* those involving the absorption or emission of a photon and/or when the kinetic energy of particles is transformed into any form of electromagnetic radiation. For example, a resonant energy transfer collision in which two atoms absorb or emit photons during the collision or the emission/absorption of a photon by an atom or molecule.

Plasma particles, ions electrons and neutral atoms exchange energy through a number of collisional processes and the rate of this transfer depends on the physical characteristics of the molecular encounter. For example, the calculation in box 2.1 shows how the kinetic energy exchange between two particles of masses m_a and m_b in an elastic collision depends on their relative masses.

2.2 Collision length and time scales

Our next step is to develop an elementary description for atomic and molecular encounters to evaluate the average length of the straight paths of the neutral gas test particle in figure 2.1(a). The following simple model applies to *short-range* encounters between neutral atoms, ion–neutral and electron–neutral collisions. Later, in chapter 5, we will be formulate a more advanced theory as well as the *long-range* interaction between charged particles.

The *short-range* collisional interaction takes place at typical distances in the order of particle size, therefore, we can consider a collision event to take place when both molecules come into contact. The colliding particles can be approximated by spheres of radius r_a and r_b, as shown by figure 2.3, and the molecular encounter occurs when the distance d between their centers is $d \leqslant (r_a + r_b)$.

This length defines an effective interaction surface $\sigma_{ab} = \pi(r_a + r_b)^2$ denominated *total cross section*[1] indicated by the dotted circle[2] in figure 2.3 which is simply $\sigma_{aa} = \pi(2r_a)^2$ when both molecules are equal.

Figure 2.4 shows the incoming test particle a moving along the Z axis towards $N_b = n_b \times V$ of stationary target atoms with number density n_b randomly distributed

[1] As we shall see in chapter 5 the collision cross section σ characterizes the collisional process. In general this effective surface $\sigma_{ab}(|\boldsymbol{v}_a - \boldsymbol{v}_b|, \chi)$ depends on the relative speed of colliding particles $|\boldsymbol{v}_a - \boldsymbol{v}_b|$ and the geometry of the molecular encounter, but it will be approximated by a value constant in this section.

[2] The derivation is in subsection 5.3.1 in chapter 5 and appendix A.

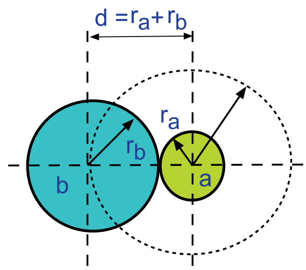

Figure 2.3. The area within the dotted circle is the total cross section σ_{ab} for colliding particles.

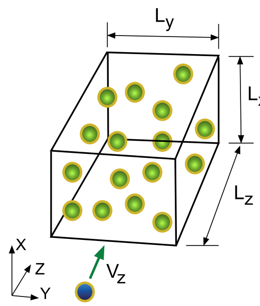

Figure 2.4. The test particle a moves into the volume occupied by target particles.

inside the volume $V = L_x\, L_y\, L_z$. The trajectory of the test particle will be blocked by targets atoms when their interaction surfaces $\sigma_{ab} \times N_b$ are equal to the cross sectional area,

$$L_x\, L_y = \sigma_{ab} \times n_b \times (L_x\, L_y\, L_z)$$

The test particle hits at least one of them when L_z is long as,

$$\lambda_c = 1/(n_b\, \sigma_{ab}) \tag{2.2}$$

The time τ_c between two successive collisions is,

$$\tau_c = \frac{\lambda_c}{v_z} = \frac{1}{\sigma_{ab}\, n_b\, v_z}$$

and the number of encounters,

$$\nu_c = \frac{1}{\tau_c} = \sigma_{ab}\, n_b\, v_z = n_b\, K \tag{2.3}$$

is the *collision frequency* which represents the number of collisional events per unit of time *experienced by the test particle*. The *reaction rate* or *rate constant* $K = \sigma_{ab}\, v_z$ is the collision frequency per unit density of target particles.

Equivalently, the mean free path $\lambda_c = 1/(n_b\,\sigma_{ab})$ is also the relation between the distance traveled by the test particle $l = v_z \times t$ and the number of collision events $(\sigma_{ab}\,n_b\,v_z) \times t$ during the same time interval. To obtain the *total number of collisions* per volume unit between a and b particles we need to multiply by the number of a particles,

$$\nu_{ab} = n_a\,n_b\,\sigma_{ab}\,v_z \tag{2.4}$$

The *mean free path* λ_c represents the average distance a particle travels between two encounters and provides the *length scale* for short range collisional interaction. In other words, the average length of the straight paths of the neutral atoms in figures 2.1(a) and 2.1(b).

The average collision time $\tau_c = 1/\nu_c$ gives the *time scale* of these *short-range* molecular encounters. The cross section in equations (2.2) and (2.3) depends on the particular characteristics of each molecular encounter (ion–neutral, electron–neutral, etc) and introduces different scales of length and time for each collisional process.

Up to this point we have not considered the fact that both target and test particles are in relative motion. The above expressions are correct insofar as the speed v_z of the test particle is much higher than the velocities of target particles.

However, in gases in thermal equilibrium the velocity of molecules has a statistical distribution. The average velocities of both colliding molecules are equal and we need to consider their statistical average[3] relative velocity $\langle \boldsymbol{v}_{rel} \rangle$. As we shall see, the rigorous calculation[4] introduces a factor $\sqrt{2}$ in the collision frequency $\nu_{ab} = n_a\,n_b\,\sigma_{ab}\,\sqrt{2}\;v_z$ mean free path $\lambda_c = 1/(\sqrt{2}\;n\,\sigma)$ for equal particles.

The characteristic distance λ_c also allows one to evaluate the depth the test particle of figure 2.4 penetrates into the gas volume. In the slab of area $S = L_x\,L_y$ and infinitesimal thickness dz of figure 2.5 the effective area occupied by target atoms is $S_t = \sigma_{ab} \times n_t \times (L_x\,L_y\,dz)$. In a molecular collision we consider the incident a particles with speed v_z are absorbed and/or rejected back by the target particles b and then they disappear from the incident flux.

The probability that an incoming particle collides on the slab is the area S_t of target molecules divided by the total area $L_x\,L_y$ of the slab,

$$P = \frac{S_t}{S} = \frac{\sigma_{ab}\,n_t\,(L_x\,L_y\,dz)}{L_x\,L_y} = \sigma_{ab}\,n_t\,dz$$

The decrease of the incoming particle flux $\Gamma_o = n_i\,v_z$ along Z is the flux $\Gamma(z)$ multiplied by the probability of being stopped by the target atoms,

$$d\Gamma = -\Gamma(z) \times (\sigma_{ab}\,n_t\,dz) = -\frac{\Gamma}{\lambda_c}$$

The integration of this ordinary differential equation gives,

[3] The statistical average over the particle speeds will be introduced in chapters 3 and 5.

[4] Discussed in section 3.4 in chapter 3.

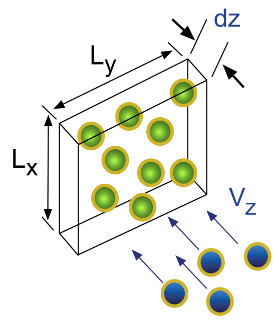

Figure 2.5. Target particles within $L_x \times L_y \times dz$.

$$\Gamma(z) = \Gamma_o e^{-z/\lambda_c}$$

This flux Γ_o of incident particles is exponentially attenuated with the rate $\lambda_c = n_t\ \sigma_{ab}$ with the depth z. Equivalently, by the total collision cross section σ for molecular encounters between incident and target particles.

The simple ideal gas law $p_a V = N\ k_B T$ describes the equilibrium states of neutral gases and moderate pressures and temperatures. It can be combined with equations (2.2) and (2.3) with $n_t = N/V$ and for collisions with neutral atoms we have,

$$\lambda_c = \left(\frac{k_B}{\sigma_{ab}}\right) \times \frac{T}{p_a} \quad \text{and,} \quad \nu_c = \left(\frac{\sigma_{ab}}{k_B}\ v_z\right) \times \frac{p_a}{T}$$

where v_z is the characteristic speed of incoming particles in figure 2.5. In a gas in thermal equilibrium the colliding particles are equal, and this speed is $\bar{v} = (8k_B T/\pi m)^{1/2}$ the average velocity of molecules[5].

The product $\lambda_c \times p \propto C(T)$ is a temperature dependent constant that can be determined in the laboratory. The experimental values for $C(T)$ for $T = 273.15$ K (0 °C) for several neutral gases are in table 2.1, the corresponding mean free paths and collision frequencies are represented in figure 2.6. The product $\lambda_c \times p$ is also estimated in the example of box 2.2 using the atomic cross section for argon as well as the collision frequencies and mean free paths.

Table 2.1. The $\lambda_c \times p$ values at 0 °C for selected gases from reference [1].

Gas	Chemical symbol	$\lambda_c \times p$ m mbar	$\lambda_c \times p$ m Pa
Hydrogen	H_2	11.5×10^{-5}	11.5×10^{-3}
Nitrogen	N_2	5.9×10^{-5}	5.9×10^{-3}
Helium	He	17.5×10^{-5}	17.5×10^{-3}
Argon	Ar	6.4×10^{-5}	6.4×10^{-3}
Xenon	Xe	3.6×10^{-5}	3.6×10^{-3}
Air		6.7×10^{-5}	6.7×10^{-3}

[5] This *mean thermal speed speed* will be introduced in section 3.3, equation (3.13) in next chapter.

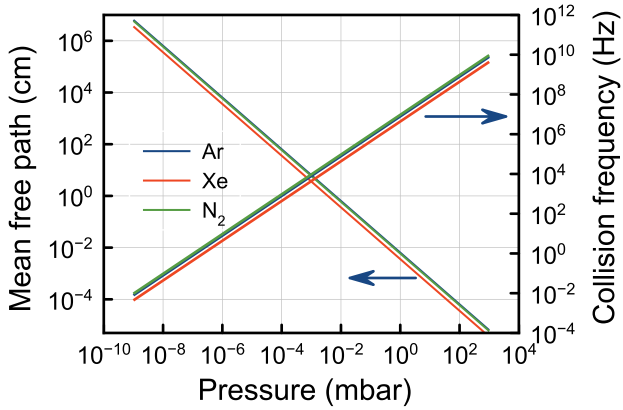

Figure 2.6. The mean free path λ_c (left) and the collision frequency ν_c (right) as a function of the gas pressure p_a calculated using the values of table 2.1.

Box 2.2. Calculation of mean free paths and collision frequencies

The *Van de Waals radius* $R_{Ar} = 1.88 \times 10^{-10}$ m of argon represents the closest approach to another atom considered as a hard sphere. Its total cross section for collisions between two neutral atoms is,

$$\sigma = \pi(2\,R_{Ar})^2 \simeq 4.44 \times 10^{-19}\,\mathrm{m}^2 \quad \text{and this gives,}$$

$$\lambda_c = \frac{k_B\,T}{\sigma\,p\,\sqrt{2}} = 2.2 \times 10^{-5} \times \frac{T\,[\mathrm{K}]}{p\,[\mathrm{Pa}]}$$

For 273.15 K (0 °C) we obtain $\lambda_c \times p = 6.02 \times 10^{-3}$ m Pa and the correct value in table 2.1 is 6.4×10^{-3}. For the pressure of $p = 1$ mbar (100 Pa) we obtain $\lambda_c = 6.4 \times 10^{-5}$ m that increases to $\lambda_c = 0.064$ m for $p = 10^{-3}$ mbar (0.1 Pa).

The calculation of the collision frequencies $\nu_c = v_z/\lambda_c$ in figure 2.6 makes use of the speed $v_z = \bar{v} = (8k_BT/\pi m)^{1/2}$ at 273 K. For argon molecules it is 506 m s^{-1}, in xenon 280 m s^{-1} and for N_2 603 m s^{-1}.

With this speed for argon atoms, at $p = 1$ mbar we have $\nu_c = 9.4 \times 10^7$ s^{-1} (9.4 MHz) and for $p = 10^{-3}$ mbar we obtain $\nu_c = 9.4 \times 10^3$ s^{-1} (9.4 KHz).

As shown in figure 2.6, for a fixed gas temperature the collision frequency increases with the neutral gas pressure, whereas the mean free path is inversely proportional to it. The distance between two successive molecular encounters grows and the number of collisions decreases as the gas pressure reduces.

The total cross section of each collisional process determines λ_c and ν_c, which could be different and also depend on the density of target particles n_t. According to the degree of ionization α_g of the gas, the length covered by a neutral between two encounters with ions could be much longer than that corresponding to a collision between two neutrals. Equivalently, the number of collisions with neutrals can be higher than a molecular encounter with ions. The gas temperature increases the mean kinetic energy of molecules and also affects the collision rate.

The *long-range* electromagnetic forces acting between electrons and ions decrease slowly with the distance between particles. However, the mixture of positive and negative particles screens the electric fields along a characteristic distance inside the plasma that allows defining an average collisional mean free path. As we shall see in chapter 5, this long-range Coulomb interaction also introduces collisional lengths (λ_{ee}, λ_{ei}, λ_{ii}) and time scales (τ_{ee}, τ_{ei}, τ_{ii}) for the interaction between the charged species.

2.3 The plasma state of condensed matter

The collision frequencies $\nu_c = 1/\tau_c$ and mean free paths λ_c for short and long-range collisions provide us with an estimate of the fundamental time and length scales for a gas with an appreciable $\alpha_g \leqslant 1$ ionization degree, or equivalently, for a plasma in stationary equilibrium. The characteristic size of the system L_s needs to be much larger $L_s \gg \lambda_c$ than all collisional mean free paths and the time scale of the macroscopic physical magnitudes τ_s much slower $\tau_s \gg \tau_c$ than characteristic collision times. Both requisites involve sufficiently high collision frequencies (equation (2.4)) per unit volume, which are proportional to the plasma particle densities n_a, n_i and n_e or equivalently, a huge number of molecular encounters during the time τ_s within the volume L_s^3.

It is not possible to formulate a simple statement that defines all possible physical systems that can be classified as a plasma, however, we can say the following.

The *plasma state of condensed matter* is a mixture of positively charged particles, electrons and neutral atoms that constitutes a *macroscopic electrically neutral medium that responds to the electric and magnetic fields in a collective mode.*

This definition will be refined later in chapter 4 by introducing the concepts of Debye length, and ion and electron plasma frequencies that consider the electromagnetic interaction and will provide us with additional plasma length and time scales.

In *weakly ionized plasmas* where $\alpha_g \ll 1$ the collision frequencies (2.4) ν_{ie}, nu_{ii} and ν_{ee} between charged particles,

$$\nu_{ia} \simeq \nu_{ea} \gg \nu_{ee}, \nu_{ii}, \nu_{ie}$$

are much lower than those between neutrals with ions ν_{ia} and electrons ν_{ea}. The opposite occurs in *strongly ionized* plasmas ($\alpha_g \lesssim 1$) where collisions between charged particles are dominant.

The physical systems that comply with the above statement are very diverse. For gas pressures close to the atmospheric conditions, *short-range* collisional mean free paths are in the order of 10^{-4}–10^{-5} cm and therefore the condition $\lambda_c \ll L$ is fulfilled by volumes $V \sim L_s^3$ well below the cubic centimeter. However, astrophysical plasmas can also meet this criterion because, despite the low particle densities giving long collisional mean free paths, the characteristic length L_s of the system can be in the order of interplanetary distances.

Plasmas could have very dissimilar compositions and structures and are chemically active media because ions can react with neutral particles or other ions. *Multicomponent plasmas* are constituted by a mixture of different kinds of atoms, ions and/or molecules with negative electric charge. These negatively charged ions can be also be produced by electron attachment in electronegative gases such as O_2 or NO_2. The *dusty plasmas* also have solid microparticles that acquire (positive or negative) electric charge in addition to electrons and ions. In these physical systems, the large mass and huge electric charge acquired by these dust grains introduce new time scales and additional physical properties.

The kinetic energy per molecule $\langle m\mathbf{v}^2/2\rangle \sim T$ is related to the temperature of the system, that must be high to sustain the ionization rate $\alpha_g > 0$. Assuming that the ions and neutral atoms are in their ground state, the Saha equation relates the electron n_e, ion n_i and neutral n_a densities of the above electron impact ionization reaction,

$$\frac{n_e\, n_i}{n_a^2} \sim \frac{(2\pi\, m_e k_B T)^{3/2}}{h^3} e^{-E_i/k_B T}$$

where E_i is the ionization energy and h the Plank's constant. To maintain an appreciable degree of ionization α_g this equation needs temperatures well over the critical point and therefore, the plasma thermodynamic equilibrium states in the schemes of figures 1.1(a) and 1.1(b) are limited to high temperatures.

As we shall see in chapter 6, a large number of different elementary processes are produced in plasmas at the molecular and atomic levels that involve charge production, excitation, light emission, etc. The thermodynamic equilibrium of a plasma is characterized by the detailed balance of each elementary process at the atomic level with its reverse molecular encounter. Such a state of equilibrium is seldom found in nature and most plasmas are physical systems *far from thermodynamic equilibrium*.

The observation of a large part of matter in the Universe in ionized state justifies the assertion that plasmas constitute a *fourth state of matter* in addition to the solid, liquid and neutral gas in figure 1.1(b). However, *most plasmas that we find in nature and in the laboratory are not in thermodynamic equilibrium* but in a *stationary equilibrium* instead, where energy and particles are lost by different physical mechanisms (such as the emission of visible light, charge losses, electric current transport, etc). Losses are balanced with other external sources of energy, for example, the ions and electrons recombine on the plasma exposed surfaces but meanwhile new charged particles are produced by an external flux of ionizing electrons, ultraviolet radiation, etc. These unavoidable losses of energy and/or particles are compensated by an external agent that maintains the steady equilibrium.

2.4 Additional considerations

- For simplicity, we will limit ourselves in the following to classical plasmas composed by electrons and single charged ions. Except when explicitly mentioned we will not consider gravitational forces in the following. On the laboratory length scale the effect of Earth's gravitational attraction on the trajectories of atoms or molecules is negligible.
- The electromagnetic interaction will be considered instantaneous (we will not consider relativistic effects) and the time variations of electromagnetic fields will be much slower than the characteristic plasma time scale that we will introduce later in chapter 4. Furthermore, the wavelength λ_{em} of the electromagnetic radiation is much longer than the characteristic size $L \ll \lambda_{em}$ of the system. The electromagnetic force acting on charged particles is,

$$\boldsymbol{f}_L = q_\alpha(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \tag{2.5}$$

where $q_\alpha = \pm e$ is the electric charge for single charged ions or electrons ($\alpha = i, e$). The variation in time of electromagnetic fields will be much slower than characteristic times involved in the collisional processes or molecular motion. We will make frequent use of Maxwell equations (2.6) in vacuum,

$$\nabla \cdot \boldsymbol{E} = \frac{\rho_c}{\varepsilon_o} \qquad \text{Gauss's law} \tag{2.6a}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad \text{Faraday induction law} \tag{2.6b}$$

$$\nabla \cdot \boldsymbol{B} = 0 \qquad \text{Gauss's law form magnetism} \tag{2.6c}$$

$$\nabla \times \boldsymbol{B} = \mu_o\left(\boldsymbol{J} + \varepsilon_o \frac{\partial \boldsymbol{E}}{\partial t}\right) \qquad \text{Ampere–Maxwell law} \tag{2.6d}$$

and $c^2 = 1/\mu_o\, \varepsilon_o$ is the speed of light. In this slow time scale the electric field is $\boldsymbol{E}(\boldsymbol{r}, t) = -\nabla\, \phi(\boldsymbol{r}, t)$, where $\phi(\boldsymbol{r}, t)$ is the electric potential and $\boldsymbol{B}(\boldsymbol{r}, t)$ the magnetic induction. The electric charge density and electric current are, respectively, $\rho_c(\boldsymbol{r}, t)$ and $\boldsymbol{J}(\boldsymbol{r}, t)$.

2.5 Commentaries and further reading

The neutral gas pressure range in figure 2.6 is roughly divided into *ultrahigh vacuum* (below 10^7 mbar) *high* (10^{-7}–10^{-3} mbar), *medium* (10^{-3}–1 mbar) and *rough* (1–10^3 mbar). The vacuum nomenclature and the definition of different pressure regimes, such as viscous, transition and molecular regime are discussed in the first two chapters in [2]. Page 8 contains a complete glossary of vacuum