Máster Universitario en Ingeniería Aeronáutica

The Space Environment

Physical description of collisions

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POLITÉCNICA

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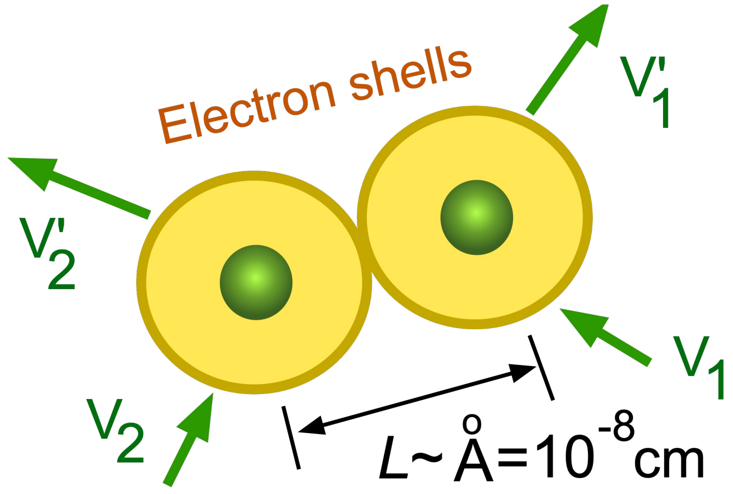
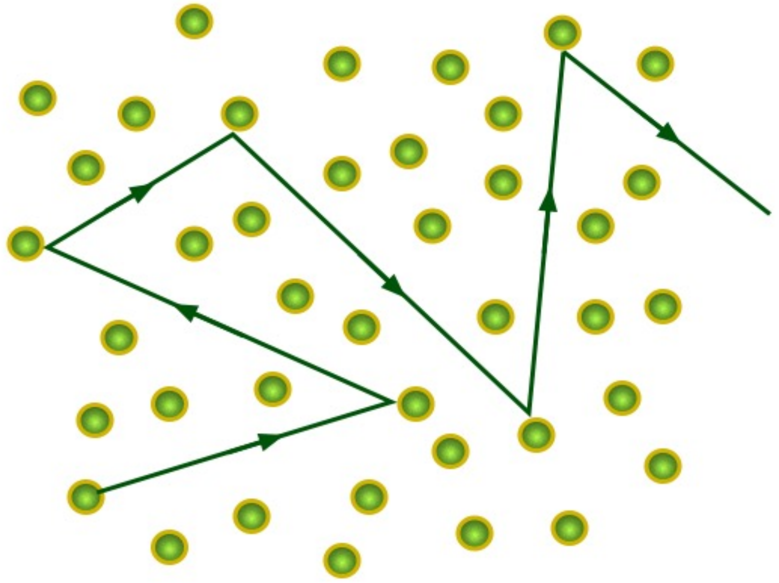
Physical description of atomic collisions

- The neutral gas molecules can be described as point-like symmetrical force centers that exchange energy and momentum.
- The ideal gas law $p_a = n_a\, k_B T$ applies to rarefied gases (low pressures) where n_a is the neutral atom gas density
- At low gas pressures, only binary elastic collisions are considered where both energy and momentum are conserved.

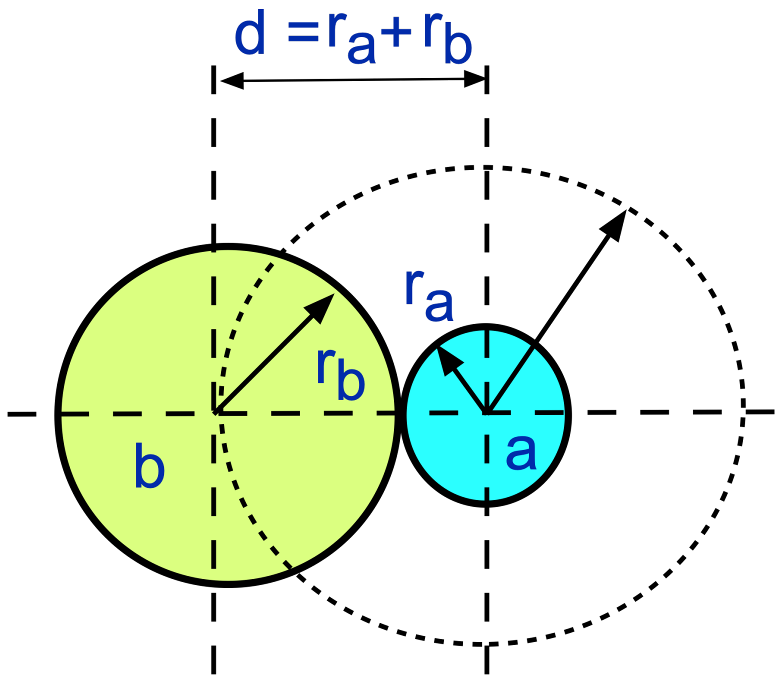


- No inter-particle electric field.
- Move randomly along straight paths.
- Neutrals interact by short-range collisions.

The motions of all atoms / molecules are independent.



Motion of a neutral gas atom



- In the hard-ball approximation (see figure) the short-range collision event between molecules of different sizes $(r_a\,, r_b)$ takes place only when,

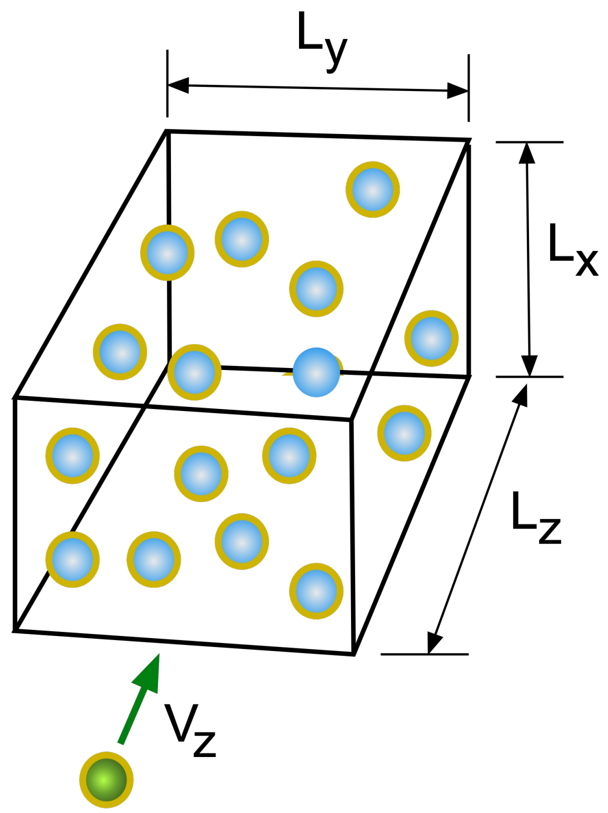
$$d \leq \ (r_a + r_b)$$

- We can introduce the effective total collision cross section as,

$$\sigma_{ab} = \pi\ (r_a + r_b)^2$$

- Cross section characterizes the effective interaction surface of a molecule is a key property that influences the drag on wind turbines, the rate of chemical reactions, etc.

Mean free path and collision frequency



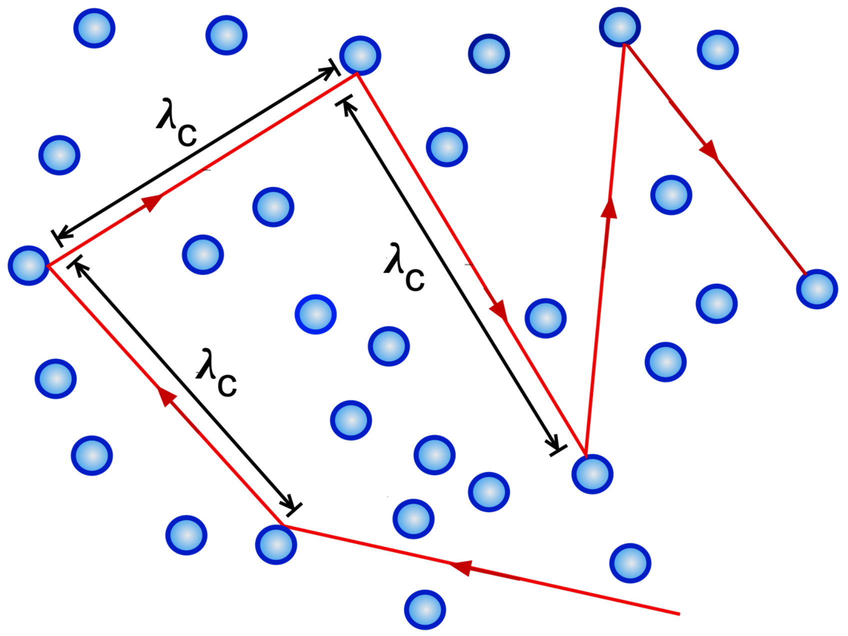
- Targets "b" (blue) are stationary in the frame where atom "a" (green) moves with velocity v_z
- Volume $V = L_x \times L_y \times L_z$
- Target atom density n_b and total targets $N_b = n_b \times V = n_b \times (L_x L_y L_z)$
- The motion of incoming (green) particle is blocked when $L_x \times L_y = n_b \times \sigma_b \times (L_x L_y L_z)$

The maximum penetration depth L_z gives the collision mean free path: $$\lambda_{ab} = \frac{1}{n_b \times \sigma_{ab}}$$

The motion of the incoming particle is random walk made of straight paths and the collisional mean free path is the average distance it travels between two successive collisions. The time elapsed between two consecutive encounters gives the collision frequency,

$$\tau_c = \frac{\lambda_{ab}}{v_z} \qquad \nu_{ab} = \frac{1}{\tau_c} = n_a\, \sigma_{ab}\, v_z$$

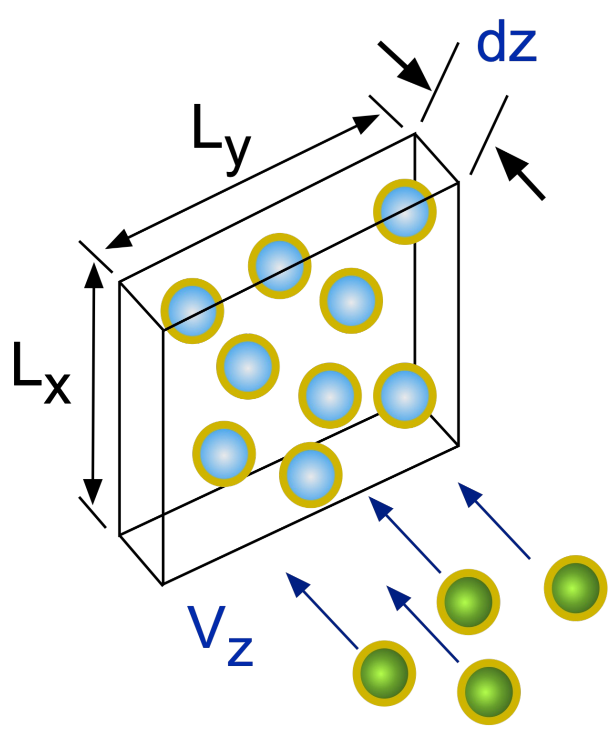
Characterizes the number of collisions by one incoming "a" particle by time unit



With the n_a the particle density we obtain the number of collision events by time and volume units between the "a" and "b" species.

$$\nu_{ab} = \nu_c \times n_a = n_a\, n_b\, \sigma_{ab}\, v_z$$

These ideas can be generalized using the probability concepts.



- Area occupied by the "b" target (blue) particles. $$A_b = \sigma_{ab}\, n_b\, (L_x \times L_y \times dz)$$
- Area available for the incoming "a" projectiles (green). $$A = (L_x \times L_y)$$

$$dP = \frac{A_b}{A} \leq 1$$

The ratio dP characterizes the probability of one "a" incoming particle hits one "b" target within the slab of dz thickness.

$$dP = \frac{\sigma_{ab}\, n_b\, (L_x \times L_y \times dz)}{L_x \times L_y} = \sigma_{ab}\, n_b\, dz \qquad \frac{dP}{dz} = \sigma_{ab}\, n_b = \frac{1}{\lambda_c}$$

The decreasing flux of incoming "a" particles along the Z depth in the figure is $\Gamma_a(z) = n_a(z)\, v_z$ with $\Gamma_a(0) = n_{ao}\, v_z$ and using the above probability of collision,

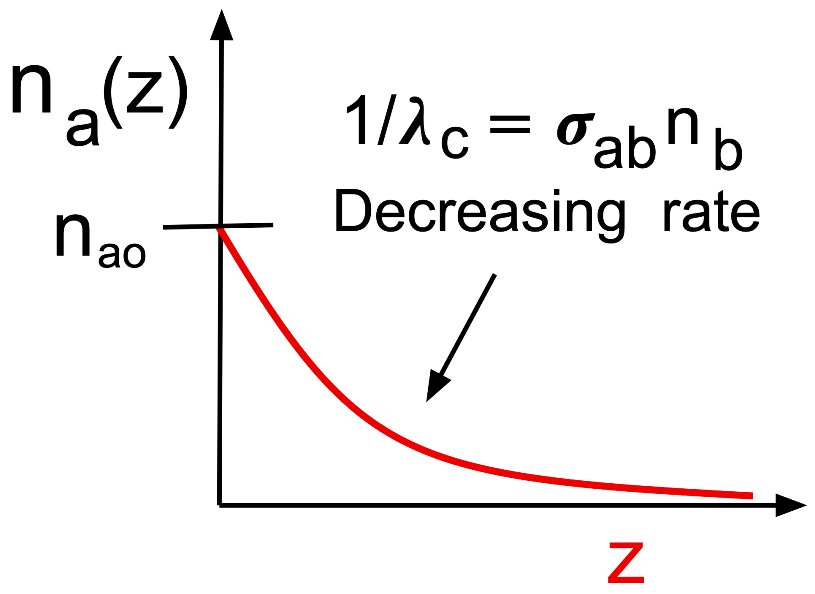
$$d\Gamma_a(z) = -\Gamma_a(z)\, dP = -\frac{\Gamma_a(z)}{\lambda_c}\, dz$$

Equivalently, particles are lost as the "a" particle beam penetrates in the volume filled with "b" targets. $$\frac{d\Gamma_a}{dz} = -\frac{\Gamma_a(z)}{\lambda_c} < 0$$

Integration gives, $$n_a(z) = n_{ao}\, e^{-z/\lambda_c}$$

Using $z = v_z\, t$ and the collision frequency $\nu_c = n_b\, \sigma_{ab}\, v_z$ we also have,

$$n_a(z) = n_{ao}\, e^{-\nu_c\, t}$$



Collision cross sections of neutral atoms

- Atomic radius of a chemical element is the distance from its center to the outermost electron shell (not well defined).
- The Van der Waals radius consider the atoms as an imaginary solid sphere and can be used to estimate the cross section.

Argon gas: Ar monotomic molecule

$$\sigma_{Ar} = \pi d^2 = \pi\,(r_{Ar} + r_{Ar})^2$$

$$\sigma_{Ar} \simeq \pi\,(2R)^2 = 3.14 \times (2 \times 188 \cdot 10^{-12})^2$$

$$\sigma_{Ar} = 4.44 \cdot 10^{-19}\ \mathrm{m}^2$$

Nitrogen gas: N_2 biatomic molecule

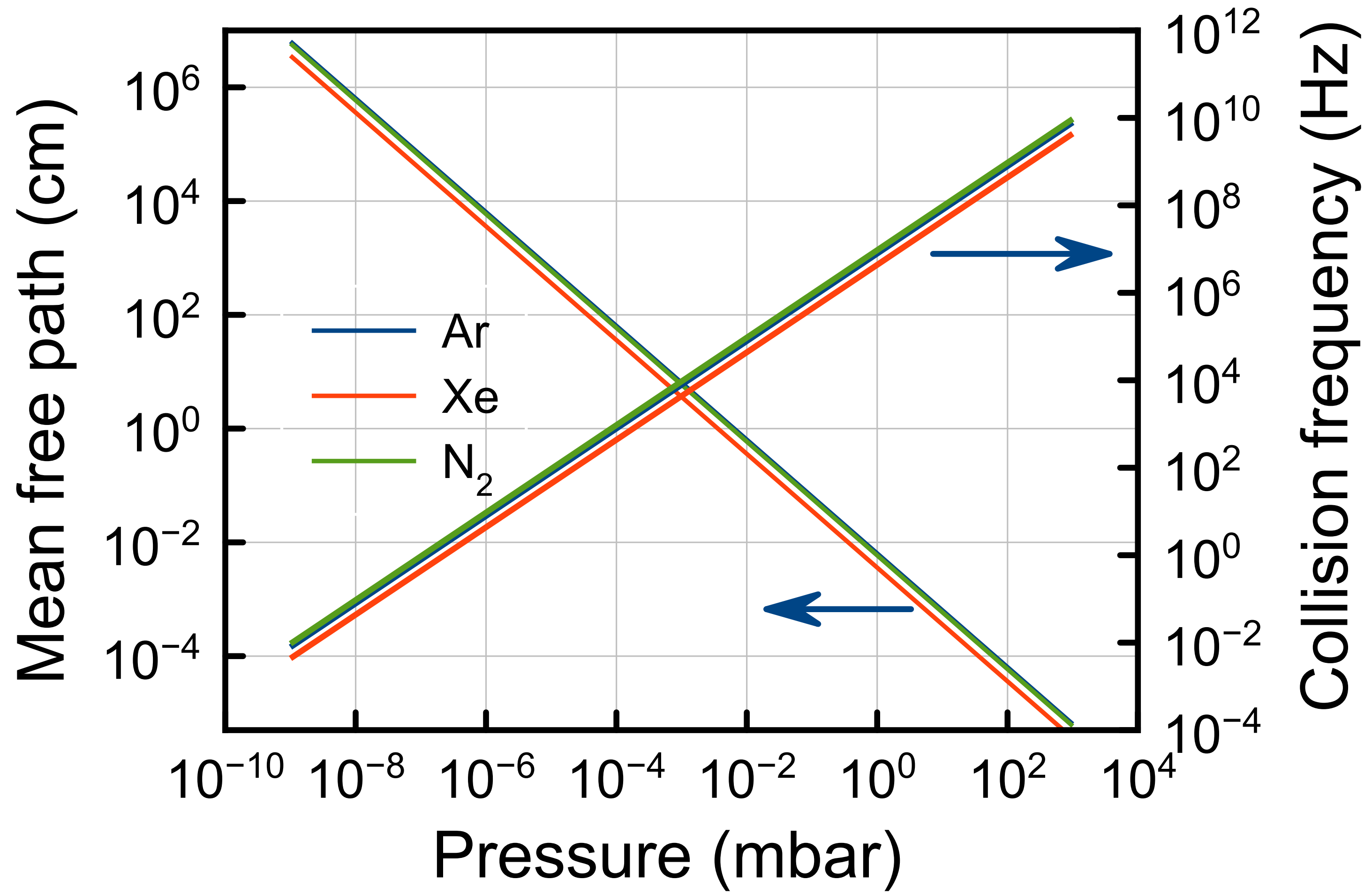
$$\sigma = \pi d^2 = \pi\,(2\,r_N + 2\,r_N)^2$$

$$\sigma_{N_2} \simeq \pi\,(4R)^2 = 3.14 \times (4 \times 155 \cdot 10^{-12})^2$$

$$\sigma_{N_2} = 1.20 \cdot 10^{-18}\ \mathrm{m}^2$$

Characteristic values using the VdW radius		
Element	r_{vW} $(\times 10^{-12})$ m	σ_{aa} $(\times 10^{-19})$ m^2
Ar	188	4.44
H	120	1.81
H_2	$2 \times 120 = 240$	7,24
He	140	2.46
N	155	3.02
N_2	$2 \times 155 = 310$	12.07
O	152	2.91
O_2	$2 \times 152 = 304$	11.61

Application: Ideal gas



For collisions between gas particles ("aa") in thermal equilibrium using the ideal gas equation $p = n_a\, k_B\, T$

$$\lambda_c = \frac{1}{\sigma_{aa}\, n_b} = \frac{k_B T}{p \times \sigma_{aa}} = C \times \left(\frac{T}{p}\right)$$

- λ_c increases with $\langle E \rangle \sim T$
- λ_c decreases with $p \sim n_a$

Argon gas: Atomic radius: $R_{Ar} \simeq 1.88 \cdot 10^{-10}$ m

$$\lambda_{Ar} = \frac{k_B\, T}{\sigma_{Ar} \times p} = \frac{1.38 \cdot 10^{-23} \times 273}{4.44 \cdot 10^{-19} \times p} = \frac{8.49 \cdot 10^{-3}}{p}$$

Table 2.1. The $\lambda_c \times p$ values at 0 °C for selected gases from reference [1].

Gas	Chemical symbol	$\lambda_c \times p$ m mbar	$\lambda_c \times p$ m Pa
Hydrogen	H_2	11.5×10^{-5}	11.5×10^{-3}
Nitrogen	N_2	5.9×10^{-5}	5.9×10^{-3}
Helium	He	17.5×10^{-5}	17.5×10^{-3}
Argon	Ar	6.4×10^{-5}	6.4×10^{-3}
Xenon	Xe	3.6×10^{-5}	3.6×10^{-3}
Air		6.7×10^{-5}	6.7×10^{-3}

The result of this simple calculation is quite like the actual value in the table.

$$\lambda_c \times p = 8.49 \cdot 10^{-3}$$

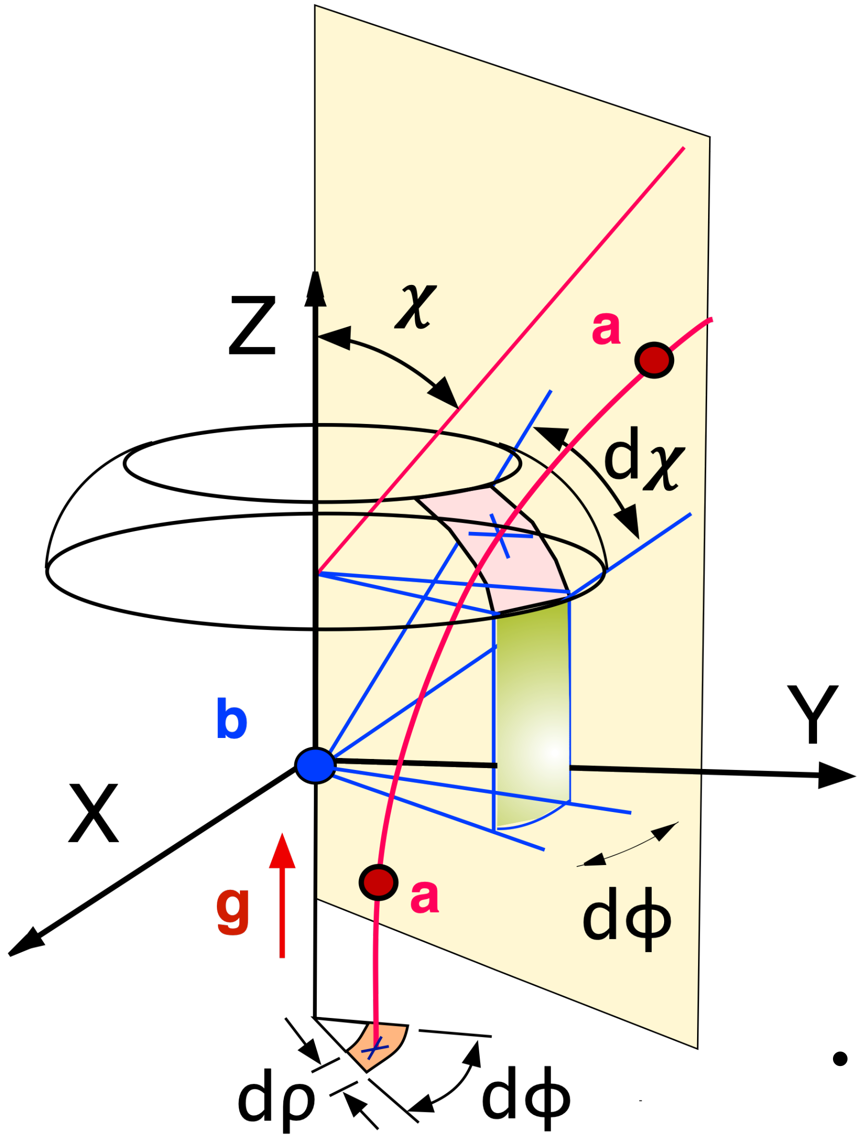
Generalization: Total and differential cross sections

- The number of collisions by time unit with one target b atom (density n_b) by the monoenergetic flux $\Gamma_a = n_a v_z$ of test particles is,

$$\dot{Q}_{ab} = \frac{\nu_{ab}}{n_b} = \frac{n_a n_b \sigma_{ab} v_z}{n_b} = \sigma_{ab}\,(n_a v_z) = \sigma_{ab}\,\Gamma_a$$

and we can alternatively introduce, $\sigma_{ab} = \frac{\dot{Q}_{ab}}{\Gamma_a}$

- Then σ_{ab} is the ratio between the number of collision events per target atom and the flux Γ_a of incoming particles.
- This concept of total cross section is valid for other processes where a physical magnitude is scattered by a target.
- The differential cross section accounts for the χ angular dependence (not dependent on ϕ by symmetry).



The $\dot{Q}_a = \Gamma_a\, dS$ particles "a" (by time unit) crossing section $dS = (\rho\, d\phi) \times d\rho$ with velocity $\boldsymbol{g} = \boldsymbol{v}_a - \boldsymbol{v}_b$

After colliding with one "b" target

$\dot{Q}_{ab}(g,\chi) = \sigma_{ab}(g,\chi)\,\Gamma_a$ particles (by time unit) are scattered into section $d\Omega = (\mathrm{r}\sin\chi\, d\phi)\times d\chi$ with velocity $\boldsymbol{g}' = \boldsymbol{v}'_a - \boldsymbol{v}'_b$

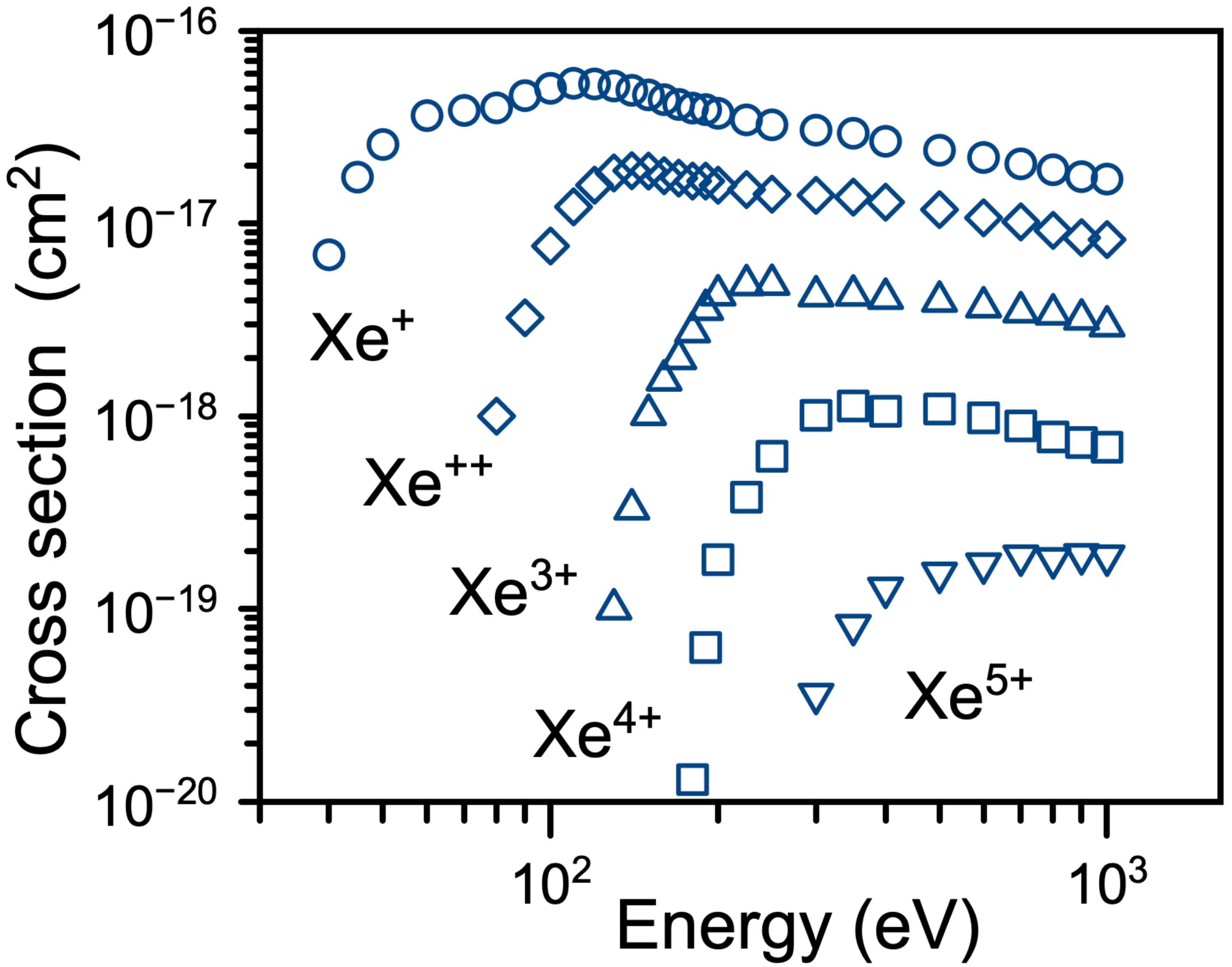
$$\sigma_{ab}(g,\chi) = \frac{\dot{Q}_{ab}(g,\chi)}{\Gamma_a}$$

$$\dot{Q}_{ab}(g) = \Gamma_a \times \int d\Omega = \Gamma_a \times \left[\int_0^{2\pi} d\phi \int_0^{\pi} \sigma_{ab}(g,\chi)\sin\chi\, d\chi\right]$$

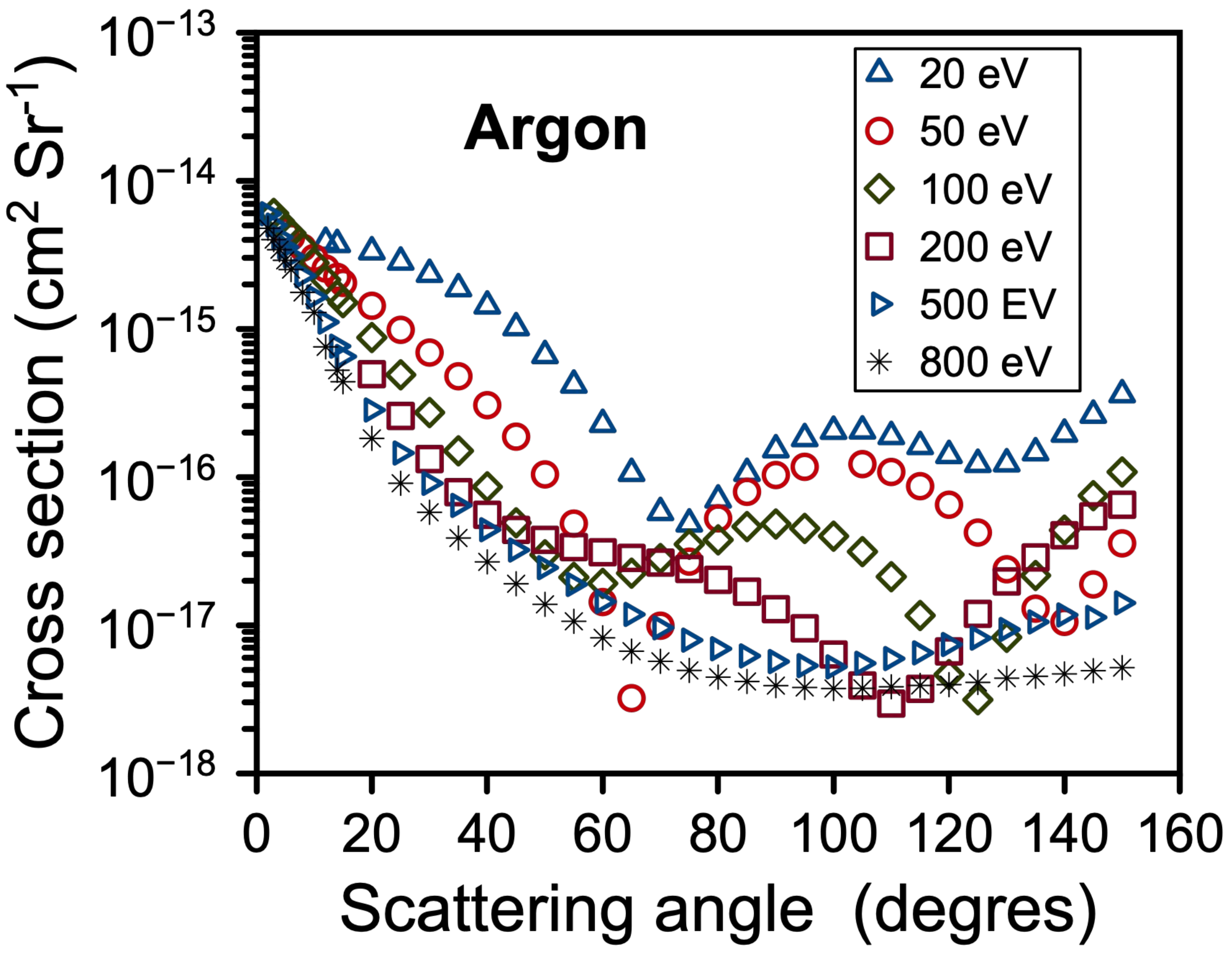
- The total cross section is therefore, $\sigma_{ab}(g) = 2\pi \int_0^{\pi} \sigma_{ab}(g,\chi)\sin\chi\, d\chi$

Examples

- Each collisional process has its characteristic cross section.
- The cross section usually depends on the energy of colliding particles.



The total ionization cross section of Xenon by electron impact $X_e + e \rightarrow X_e^n + ne$ of interest in plasma propulsion as this gas is used as propellant due to its low ionization energy.



The differential cross section of elastic scattering $Ar + e \rightarrow Ar + e$ of electrons by Argon atoms. The values depends on the energy and the angle of incidence of the electron.

Radar cross section

The radar cross section (RCS) of a target is the ratio of the power scattered back P_{Rx} to the radar receiver over the incident radar power density P_{Tx} per unit of solid angle on the target as if the radiation were isotropic.

$$\sigma_{RCS} = \lim_{r\to\infty} \frac{(4\pi r^2)\, P_{Rx}}{P_{Tx}}$$

- r is the distance between the radar and the target.
- P_{Tx} is the electromagnetic power received by the target (W/m^2).
- P_{Rx} is the electromagnetic power scattered by the target (W/m^2).

$$\sigma_{RCS} = \frac{\text{Energy at the receiver by time unit}}{\text{Energy over the target by time unit and surface}}$$

- Geometry of the target.
- Direction of the illumination radar.
- Frequency of radar signal.
- Electrical properties of the target surface.

$$\sigma_{ab} = \frac{\dot{Q}_{ab} \text{ number of “}a\text{” particles scattered by the target by time unit}}{\Gamma_a \text{ number of incident “}a\text{” particles over the target by surface and time units.}}$$