From kinetic theory to fluid dynamics

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From kinetic theory to fluid dynamics, ...

The macroscopic fluid equations are deduced by <u>taking moments</u> of the Boltzmann Eq. and low order moments are related with the physical magnitudes. The moment $\mathbf{M}^k(\mathbf{r},t)$ is the tensor,

$$\boldsymbol{M}^{(k)}(\boldsymbol{r},t)=\int_{-\infty}^{+\infty}\underbrace{(\boldsymbol{v}\otimes\ \boldsymbol{v}\otimes\ldots\otimes\boldsymbol{v})}_{\text{k times}}g(\boldsymbol{v},\boldsymbol{r},t)d\boldsymbol{v}$$

where $g(\boldsymbol{v},\boldsymbol{r},t)$

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$$\left.\begin{aligned} <H_\alpha(\boldsymbol{v},\boldsymbol{r},t)> &= \int_{-\infty}^{+\infty} H_\alpha(\boldsymbol{v},\boldsymbol{r},t)\ \hat{f}_\alpha(\boldsymbol{v},\boldsymbol{r},t)d\boldsymbol{v} \\ <H_\alpha(\boldsymbol{v},\boldsymbol{r},t)> &= \frac{1}{n_\alpha}\int_{-\infty}^{+\infty} H_\alpha(\boldsymbol{v},\boldsymbol{r},t)\ f_\alpha(\boldsymbol{v},\boldsymbol{r},t)d\boldsymbol{v} \end{aligned}\right\}$$

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and with $H_\alpha(\mathbf{v},\mathbf{r},t)=\mathbf{v}$ $\quad d\boldsymbol{\Gamma}_\alpha(\boldsymbol{r},t)=n_\alpha(\boldsymbol{r},t)\,\boldsymbol{u}_\alpha(\boldsymbol{r},t)=\int_{-\infty}^{+\infty}\boldsymbol{v}\,f_\alpha(\boldsymbol{v},\boldsymbol{r},t)d\boldsymbol{v}$

The second order moments are related with the flux of momentum (stress tensor)

$$\tilde{\boldsymbol{H}}_\alpha(\boldsymbol{v}, \boldsymbol{r}, t) = m_\alpha \left(\boldsymbol{v} \otimes \boldsymbol{v}\right) \qquad \tilde{\boldsymbol{S}}_\alpha(\boldsymbol{r}, t) = \int_{-\infty}^{+\infty} m_\alpha (\boldsymbol{v} \otimes \boldsymbol{v}) f_\alpha(\boldsymbol{v}, \boldsymbol{r}, t) d\boldsymbol{v}$$

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$$H_\alpha(\boldsymbol{v}, \boldsymbol{r}, t) = \frac{m_\alpha}{2} \ (\boldsymbol{v} \bullet \boldsymbol{v}) = \frac{m_\alpha v^2}{2} \qquad E_\alpha(\boldsymbol{r}, t) = \int_{-\infty}^{+\infty} \frac{m_\alpha v^2}{2} f_\alpha(\boldsymbol{v}, \boldsymbol{r}, t) d\boldsymbol{v}$$

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The velocity distribution function is not determined, and at this point we only require a bounded function for the particle velocities

$$f_\alpha(\boldsymbol{v}, \boldsymbol{r}, t) \to 0 \quad \text{when,} \quad \boldsymbol{v} \to 0$$

The averages and macroscopic and thermal speeds, ...

The particle velocity is expressed as the sum of a thermal speed **w** and the macroscopic fluid velocity $\mathbf{u}_\alpha$ (**r**, t) as,

$$\boldsymbol{v} = \boldsymbol{u}_\alpha(\boldsymbol{r}, t) + \boldsymbol{w} \qquad \alpha = e, i, a \qquad \langle \boldsymbol{v} \rangle = \boldsymbol{u}_\alpha(\boldsymbol{r}, t) + \langle \boldsymbol{w} \rangle$$

with, $< w_x > = < w_y > = < w_z > = 0$ and, $\langle \boldsymbol{w} \bullet \boldsymbol{w} \rangle = \langle w^2 \rangle > 0$

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Then, the average energy becomes,

$$E_\alpha(\boldsymbol{r},t) = \int_{-\infty}^{+\infty} \frac{m_\alpha v^2}{2} f_\alpha(\boldsymbol{v}, \boldsymbol{r}, t)\, d\boldsymbol{v} = n_\alpha \left\langle \frac{m_\alpha}{2} v^2 \right\rangle = n_\alpha \frac{m_\alpha}{2}(u_\alpha^2 + \langle w^2 \rangle) = \rho_\alpha \frac{u_\alpha^2}{2} + E_{i\alpha}$$

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This expression implies the choice of A MODEL for the distribution function !

The stress tensor, $\tilde{\boldsymbol{S}}_\alpha(\boldsymbol{r},t) = \int_{-\infty}^{+\infty} m_\alpha (\boldsymbol{v} \otimes \boldsymbol{v}) f_\alpha(\boldsymbol{v}, \boldsymbol{r},t) d\boldsymbol{v} = m_\alpha n_\alpha(\boldsymbol{r},t) \langle \boldsymbol{v} \otimes \boldsymbol{v} \rangle$

the components are,

$$\begin{pmatrix} v_x^2 & v_x v_y & v_x v_z \\ v_y v_x & v_y^2 & v_y v_z \\ v_z v_x & v_z v_y & v_z^2 \end{pmatrix}$$

$$\boldsymbol{v} = \boldsymbol{u}_\alpha + \boldsymbol{w}$$

$$\langle (u_{\alpha i} + w_i)(u_{\alpha j} + w_j) \rangle$$

$$S_{ij} = m_\alpha n_\alpha (\langle u_{\alpha i} u_{\alpha j} \rangle + \langle w_i w_j \rangle + u_{\alpha i} \langle w_j \rangle + u_{\alpha j} \langle w_i \rangle)$$

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Symmetric tensor
(scalar pressure)

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Symmetric tensor (scalar pressure)

Antisymmetric tensor (stresses)

This defines the <u>scalar pressure,</u> $\left\langle w_x^2 \right\rangle = \left\langle w_x^2 \right\rangle = \left\langle w_x^2 \right\rangle = \dfrac{\left\langle w^2 \right\rangle}{3}$

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The <u>energy transport</u> is related with ,

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and finally we obtain the vector,

$$\boxed{\boldsymbol{K}_\alpha(\boldsymbol{r},t) = E_\alpha \boldsymbol{u}_\alpha + \boldsymbol{q}_\alpha + \tilde{\mathbf{G}}_\alpha : \boldsymbol{u}_\alpha}$$

$$\boxed{\boldsymbol{q}_\alpha(\boldsymbol{r},t) = \int_{-\infty}^{+\infty} \left(\frac{m_\alpha w^2}{2}\right) \boldsymbol{w}\, f_\alpha(\boldsymbol{v},\ \boldsymbol{r},\ t) d\boldsymbol{v} = \frac{m_\alpha n_\alpha}{2} < w^2 \boldsymbol{w} >}$$

where is the heat flux density,

The continuity equation, ...

We apply the same averaging procedure directly to the Boltzman equation to derive the fluid transport equations,

$$\int_{-\infty}^{+\infty} \frac{\partial f_\alpha}{\partial t} d\boldsymbol{v} + \int_{-\infty}^{+\infty} \nabla_{\boldsymbol{r}} \cdot (\boldsymbol{v} f_\alpha) d\boldsymbol{v} + \int_{-\infty}^{+\infty} \nabla_{\boldsymbol{v}} \cdot (\frac{\boldsymbol{F}}{m_\alpha} f_\alpha) d\boldsymbol{v} = \int_{-\infty}^{+\infty} C_s(f_\alpha) d\boldsymbol{v}$$

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$$\frac{\partial}{\partial t} \int_{-\infty}^{+\infty} f_\alpha \, d\boldsymbol{v} + \nabla_{\boldsymbol{r}} \cdot \int_{-\infty}^{+\infty} (\boldsymbol{v} f_\alpha) d\boldsymbol{v} + \underbrace{\int_{s(v)} (\frac{\boldsymbol{F}}{m_\alpha} f_\alpha) \cdot d\boldsymbol{S}}_{f_\alpha \text{ is bounded} \to 0} = \int_{-\infty}^{+\infty} C_s(f_\alpha) d\boldsymbol{v}$$

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First moment of the Boltzmann collision operator.

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The averages consider an unspecified, bounded distribution function.

The fluid transport equations for a plasma, ...

From the three moments of the Botzmann equation, are deduced the transport of particles momentum and energy

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \boldsymbol{u}_\alpha) = F_\alpha - S_\alpha$$

$$\rho_\alpha \frac{D\,\boldsymbol{u}_\alpha}{Dt} = -\nabla_{\boldsymbol{r}}\, p_\alpha - \nabla_{\boldsymbol{r}} \cdot \tilde{\boldsymbol{\Pi}}_\alpha - m_\alpha \boldsymbol{u}_\alpha (F_\alpha - G_\alpha) + \boldsymbol{F}_\alpha^e + \boldsymbol{F}_\alpha^g + \boldsymbol{R}_\alpha$$

$$E_\alpha = \rho_\alpha (u_\alpha^2 / 2 + e_{i\alpha})$$

$$\frac{DE_\alpha}{Dt} = -\nabla_{\boldsymbol{r}} \cdot (p_\alpha\ \boldsymbol{u}_\alpha) + \boldsymbol{J}_\alpha \cdot \boldsymbol{E} - \nabla_{\boldsymbol{r}} \cdot \boldsymbol{q}_\alpha - \nabla_{\boldsymbol{r}} \cdot (\tilde{\boldsymbol{\Pi}}_\alpha : \boldsymbol{u}_\alpha) + Q_\alpha$$

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Therefore, we need of a model for the plasma and the cross sections to evaluate these expresions

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The transport coefficients: Closure

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The Krook relaxation model, ...

This model considers only one relaxation time scale and a fast return to the equilibrium state.

$$\begin{cases} C_s(f_\alpha) = \dfrac{df_\alpha}{dt} = -\dfrac{f_\alpha - f_{0\alpha}}{\tau_c} = \dfrac{1}{\tau_c} f_{1\alpha}(\boldsymbol{v}, \boldsymbol{r}, t_o) \\ f_\alpha(\boldsymbol{v}, \boldsymbol{r}, t) \simeq f_{0\alpha}(\boldsymbol{v}, \boldsymbol{r}, t_o) + f_{1\alpha}(\boldsymbol{v}, \boldsymbol{r}, t_o)\, e^{-t/\tau_c} \end{cases}$$

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equilibrium distribution

perturbed term

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Momentum: $\boldsymbol{R}_\alpha = \sum_\beta \boldsymbol{R}_{\alpha\beta}$ $\quad \boldsymbol{R}_{\alpha\beta} = -\boldsymbol{R}_{\alpha\beta}$ $\quad \sum_\alpha \boldsymbol{R}_\alpha = \sum_\alpha \sum_\beta \boldsymbol{R}_{\alpha\beta} = 0$

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Energy: $Q_\alpha = \sum_\beta Q_{\alpha\beta}$

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For elastic collisions of electrons:

$$\Delta\epsilon = \frac{m_e^2}{2m_a}(\Delta \boldsymbol{v}_e)^2 = \frac{m_e^2 v_e^2}{m_a}(1 - \langle \cos(\theta) \rangle) \qquad \boxed{Q_{ea} = \frac{2m_e}{m_a} \nu_m n_a \epsilon_e}$$

Useful fluid equations for space plasmas, ...

$$\begin{cases} \dfrac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \boldsymbol{u}_\alpha) = F_\alpha - S_\alpha \\ \rho_\alpha \dfrac{D\,\boldsymbol{u}_\alpha}{Dt} = -\nabla_r p_\alpha - m_\alpha \boldsymbol{u}_\alpha (F_\alpha - G_\alpha) + \boldsymbol{F}_\alpha^e + \boldsymbol{F}_\alpha^g + \boldsymbol{R}_\alpha \\ \dfrac{DE_\alpha}{Dt} = -\nabla_r \cdot (p_\alpha\ \boldsymbol{u}_\alpha) + \boldsymbol{J}_\alpha \cdot \boldsymbol{E} - \nabla_r \cdot \boldsymbol{q}_\alpha + Q_\alpha \end{cases}$$

$$(F_e = \nu_I\, n_e)$$ ...etc.

$$E_\alpha = \rho_\alpha (u_\alpha^2 / 2 + e_{i\alpha})$$

For elastic collisions:

$$\boldsymbol{R}_\alpha = \sum_\beta \boldsymbol{R}_{\alpha\beta} \qquad \boldsymbol{R}_{\alpha\beta} = -A_{\alpha\beta}(\boldsymbol{u}_\alpha - \boldsymbol{u}_\beta)$$

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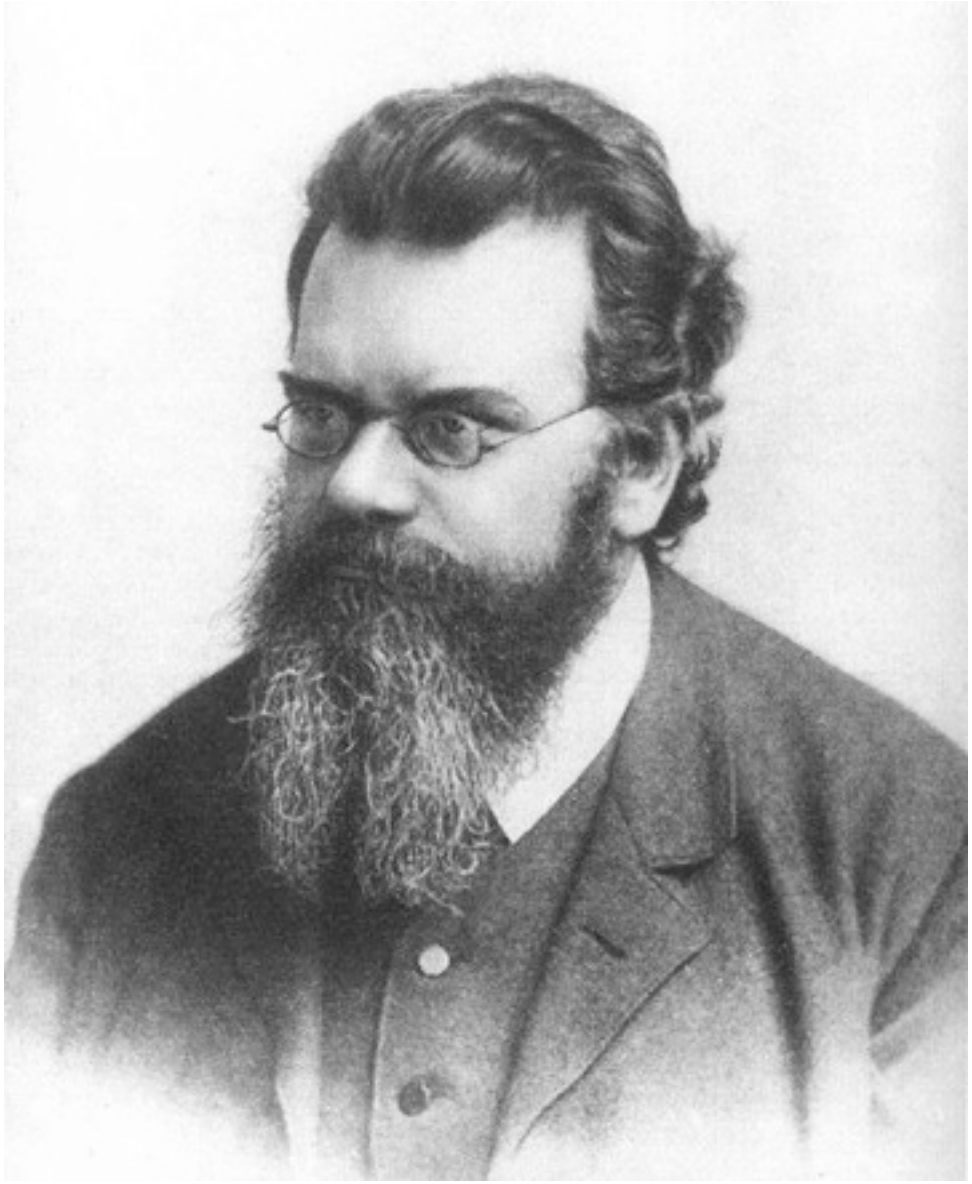
For ionizing collisions

$$\boldsymbol{R}_{Ii} = -m_i n_e \nu_I (\boldsymbol{u}_i - \boldsymbol{u}_e) \qquad \boldsymbol{R}_{Ie} = -m_i n_e \nu_I (\boldsymbol{u}_e - \boldsymbol{u}_i) \qquad Q_I = E_i\, \nu_I$$

Portrait of Ludwig Boltzmann and his grave in Zentralfriedhof, Vienna