Máster Universitario en Ingeniería Aeronáutica

The Space Environment

Plasma Parameters for the space environment applications

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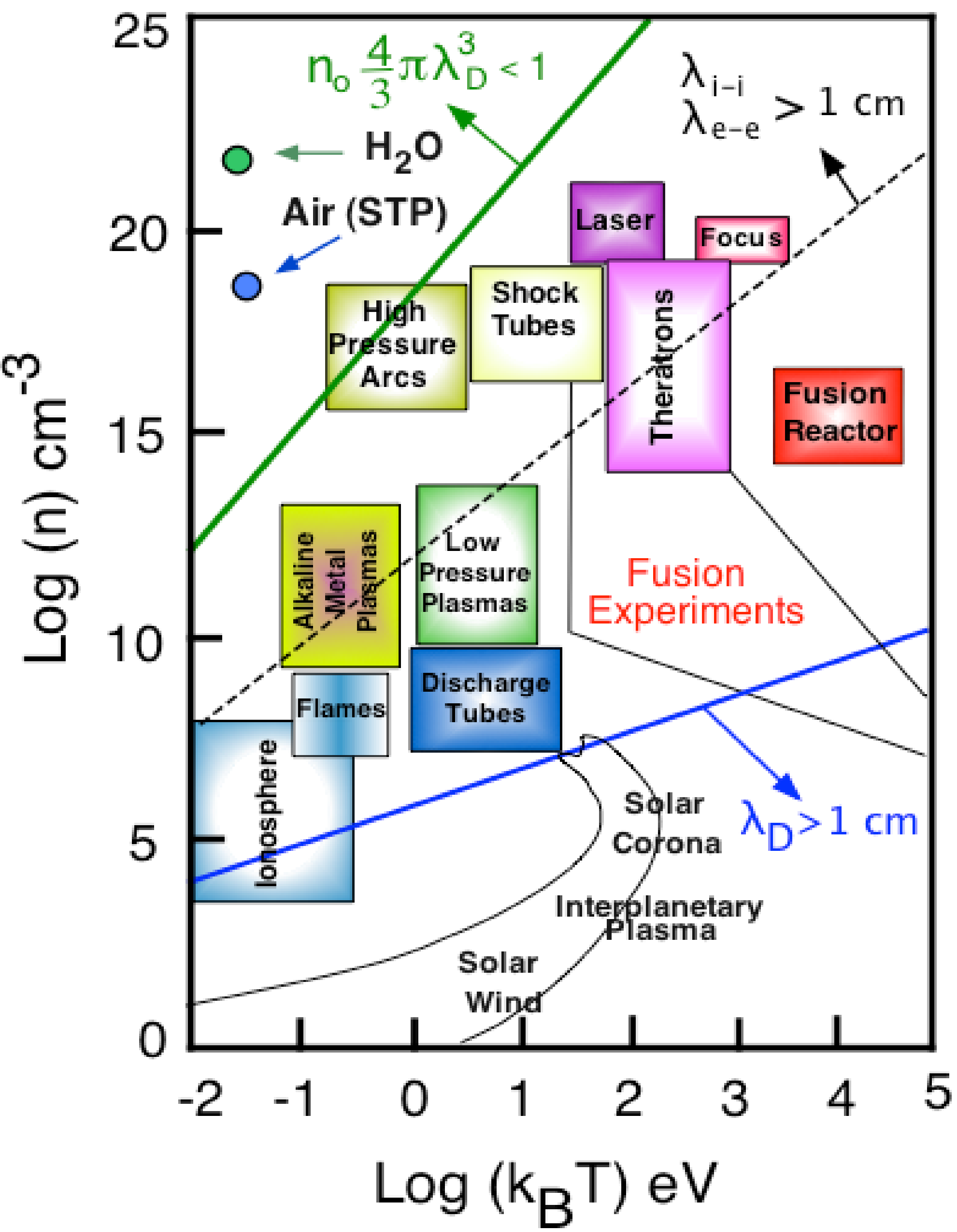
Maretial y Página web de la Asignatura basada eb ka web del Prof. Dr. L. Conde:
https://plasmalab.aero.upm.es/~lcl/EntornoEspacial/



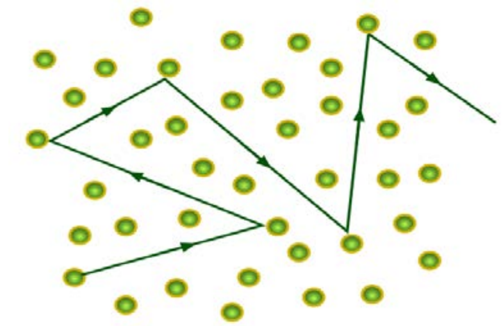
Plasmas can be roughly classified according to the number densities of charged particles and their average kinetic energies.

Room temperature: $k_B T = 0.025$ eV
- Water n $\approx 10^{22}$ cm^{-3}
- SPT Air: n $\approx 10^{19}$ cm^{-3}

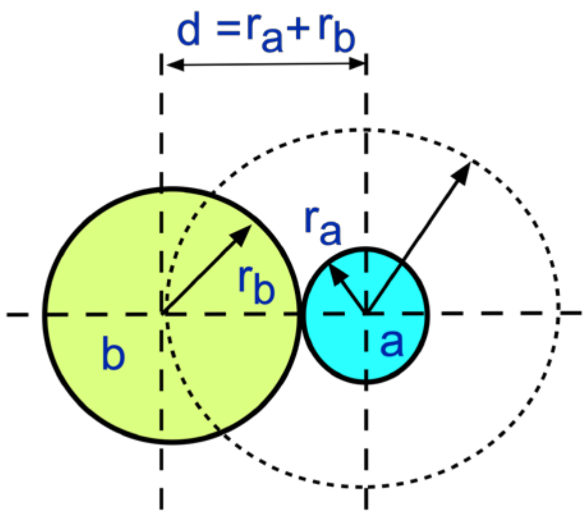
- Fusion reactor: $k_B T \approx 10^4$ eV n $\approx 10^{15}$ cm^{-3}
- Laser plasmas: $k_B T \approx 10^2$ eV n $\approx 10^{20}$ cm^{-3}
- Glow discharge: $k_B T \approx$ 1-3 eV n $\approx 10^8$ cm^{-3}
- Ionosphere: $k_B T \approx 0.05$ eV n $\approx 10^6$

collision effects

The motions of all atoms / molecules are independent.



Motion of a neutral gas atom



- We can introduce the **effective total collision cross** section as,

$$\sigma_{ab} = \pi\,(r_a + r_b)^2$$

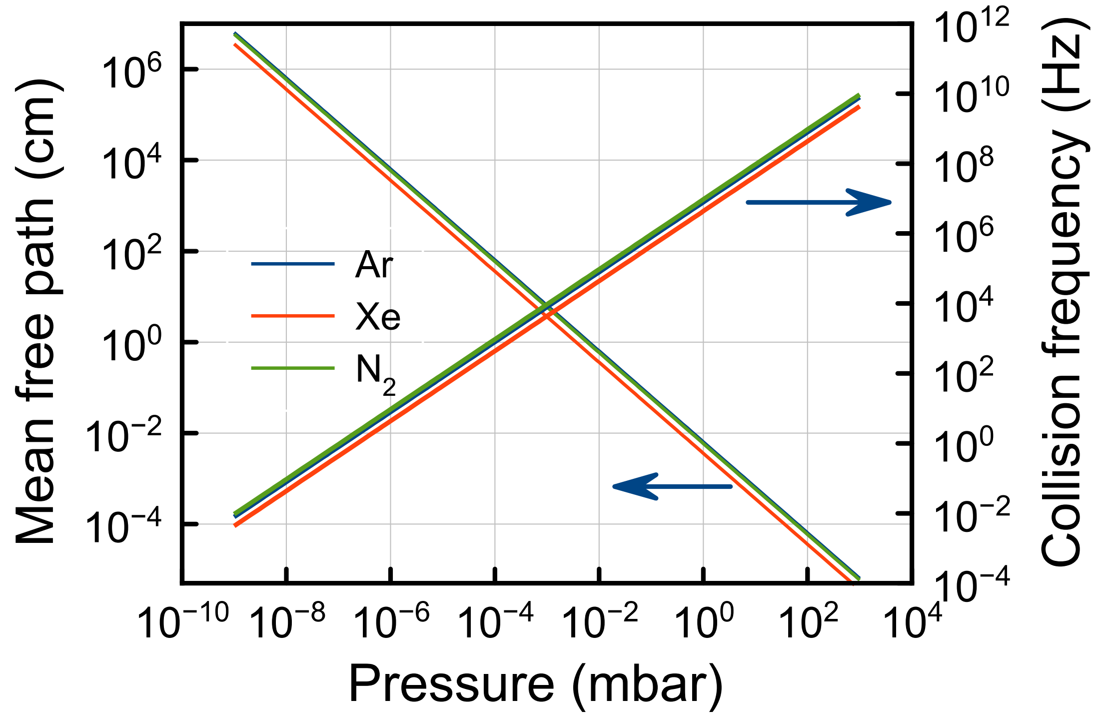
- Cross section characterizes the effective interaction surface of a molecule is a key property that influences the drag on wind turbines, the rate of chemical reactions, etc.

Relevant parameters

- Particle concentration
- Temperature
- Chemical composition

- Strong **influenced by solar activity**, $F_{10.7}$ parameter related to iron $\lambda_{Fe} = 10.7\ \mathrm{cm}$ emission associated to EUV light.
- The concentrations of all gases decrease with altitude.
- **Dissociation of molecules** produce concentrations of atomic oxygen and nitrogen that are negligible at low altitudes.
- Temperature becomes constant over 200 km.

Application: Ideal gas



For collisions between gas particles (“aa”) in thermal equilibrium using the ideal gas equation $p = n_a\, k_B\, T$

$$\lambda_c = \frac{1}{\sigma_{aa}\, n_b} = \frac{k_B T}{p \times \sigma_{aa}} = C \times \left(\frac{T}{p}\right)$$

- λ_c increases with $\langle E \rangle \sim T$
- λ_c decreases with $p \sim n_a$

Argon gas: Atomic radius: $R_{Ar} \simeq 1.88 \cdot 10^{-10}$ m

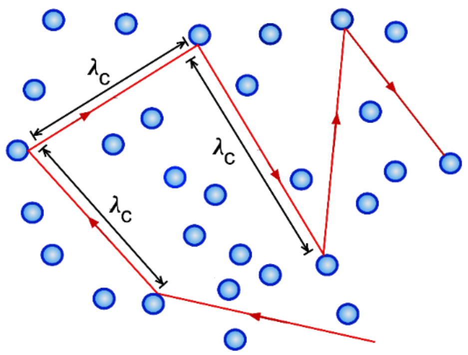
$$\lambda_{Ar} = \frac{k_B\, T}{\sigma_{Ar} \times p} = \frac{1.38 \cdot 10^{-23} \times 273}{4.44 \cdot 10^{-19} \times p} = \frac{8.49 \cdot 10^{-3}}{p}$$

The result of this simple calculation is quite like the actual value in the table.

$$\lambda_c \times p = 8.49 \cdot 10^{-3}$$

Mean free path and collision frequency

The motion of the incoming particle is random walk made of straight paths and the collisional mean free path is the average distance it travels between two successive collisions. The time elapsed between two consecutive encounters gives the collision frequency,



$$\lambda_{ab} = \frac{1}{n_b \times \sigma_{ab}} \qquad \tau_c = \frac{\lambda_{ab}}{v_z} \qquad \nu_{ab} = \frac{1}{\tau_c} = n_a\, \sigma_{ab}\, v_z$$

- Atomic radius of a chemical element is the distance from its center to the outermost electron shell (not well defined).
- The Van der Waals radius consider the atoms as an imaginary solid sphere and can be used to estimate the cross section.

Argon gas: Ar monotomic molecule

$$\sigma_{Ar} = \pi d^2 = \pi\,(r_{Ar} + r_{Ar})^2$$

$$\sigma_{Ar} \simeq \pi\,(2R)^2$$
$$= 3.14 \times (2 \times 188 \cdot 10^{-12})^2$$
$$\sigma_{Ar} = 4.44 \cdot 10^{-19}\ \mathrm{m}^2$$

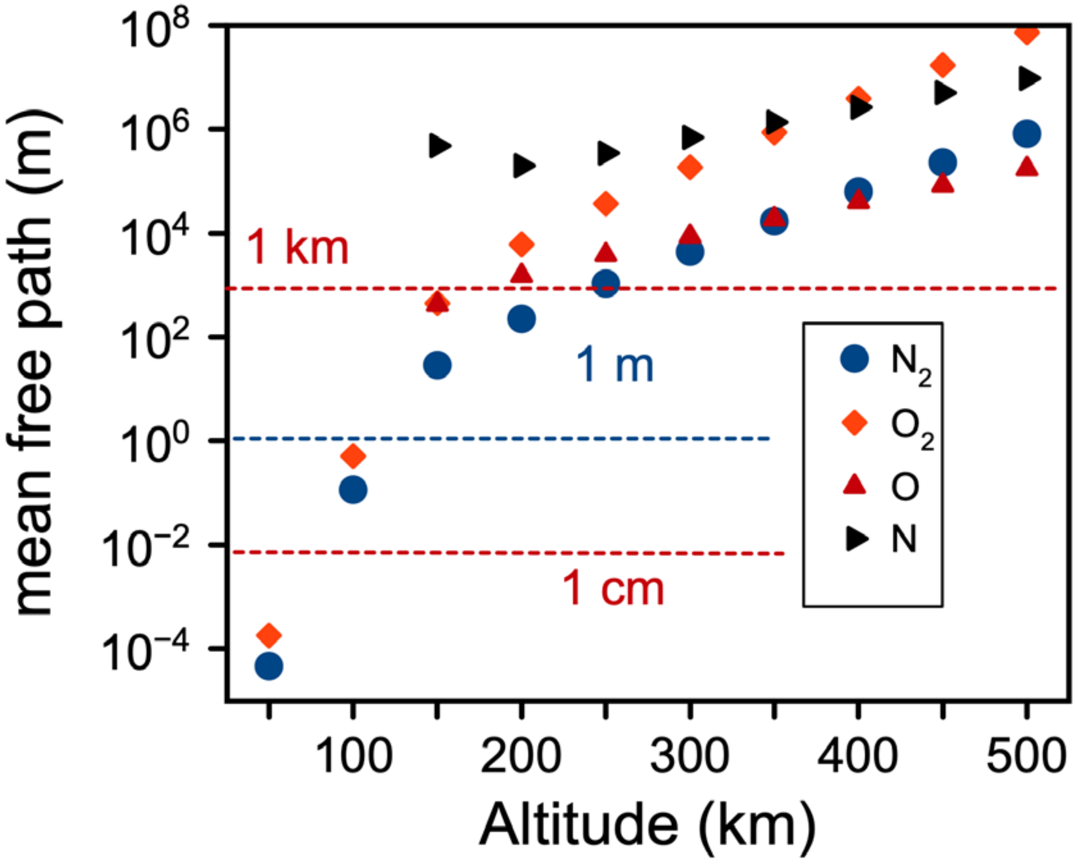
Nitrogen gas: $\mathrm{N_2}$ biatomic molecule

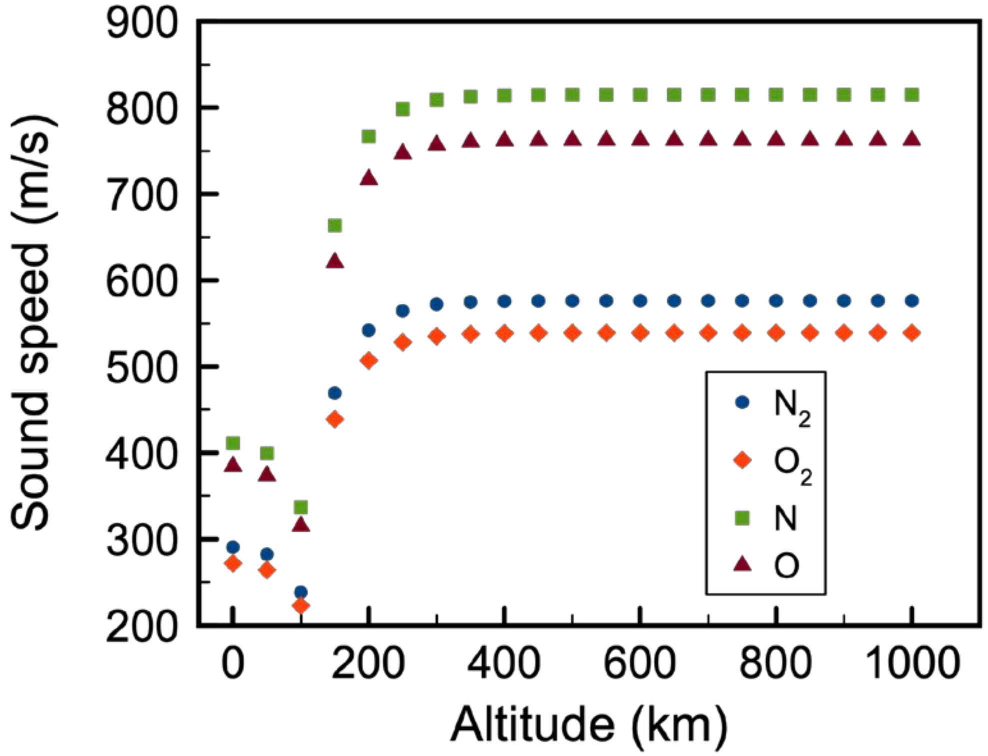
$$\sigma = \pi d^2 = \pi\,(2\, r_N + 2\, r_N)^2$$

$$\sigma_{N_2} \simeq \pi\,(4R)^2 = 3.14 \times (4 \times 155 \cdot 10^{-12})^2$$
$$\sigma_{N_2} = 1.20 \cdot 10^{-18}\ \mathrm{m}^2$$

Characteristic values using the VdW radius		
Element	r_{vW} ($\times 10^{-12}$) m	σ_{aa} ($\times 10^{-19}$) m^2
Ar	188	4.44
H	120	1.81
H_2	$2 \times 120 = 240$	7,24
He	140	2.46
N	155	3.02
N_2	$2 \times 155 = 310$	12.07
O	152	2.91
O_2	$2 \times 152 = 304$	11.61

Collision mean free path and sound speed





- The typical mfp are always much that characteristic scale $\lambda_c \gg L$ of orbital vehicles.
- Sound speed are below the typical orbital speeds of few km/s.
- Local pressures is always negligible.

- Macroscopic magnitudes are calculated as averages.
- Average kinetic energy:

$$\langle H\rangle = \int_{-\infty}^{+\infty} H(\boldsymbol{v})\, dP = \int_{-\infty}^{+\infty} H(\boldsymbol{v})\, f_{mb}(\boldsymbol{v})\, d^3v$$

$$\langle H\rangle = \int_{0}^{+\infty} H(E)\, dP = \int_{0}^{+\infty} H(E)\, g_{mb}(E)\, dE$$

$$\langle E\rangle = \int_{-\infty}^{+\infty} \frac{m\boldsymbol{v}^2}{2}\, dP = \int_0^{+\infty} E\, g_{mb}(E)\, dE = \frac{3}{2} k_B T$$

$$\langle v_x\rangle = \int_{-\infty}^{+\infty} v_x\, f_{mb}(\boldsymbol{v})\, d^3v = 0 \qquad \langle v_x\rangle = \langle v_y\rangle = \langle v_z\rangle = 0$$

- Root mean square (rms): $v_{rms} = \sqrt{\langle v^2\rangle}$

$$\langle v^2\rangle = \int_{-\infty}^{+\infty} v^2\, f_{mb}(\boldsymbol{v})\, d^3v = \int_0^{+\infty} v^2\, f_{mb}(\boldsymbol{v})\, (4\pi v^2)\, dv = \frac{3k_BT}{m}$$

- Mean thermal speed:

$$\bar{v} = \langle|\boldsymbol{v}|\rangle = \int_{-\infty}^{+\infty} v\, f_{mb}(\boldsymbol{v})\, d^3v = \int_0^{+\infty} v\, f_{mb}(\boldsymbol{v})\, (4\pi v^2)\, dv = \left(\frac{8k_BT}{\pi\, m}\right)^{1/2}$$

$$\langle|v_x|\rangle = \int_{-\infty}^{+\infty} |v_x|\, f_{mb}(\boldsymbol{v})\, d^3v = 2\int_0^{+\infty} v_x\, f_{mb}(\boldsymbol{v})\, d^3v = \left(\frac{2k_BT}{\pi\, m}\right)^{1/2}$$

$$\Gamma_x = n_o \frac{\langle|v_x|\rangle}{2} = \frac{1}{4} n_o \bar{v}$$

Particle flux over any surface exposed to the gas.

Calculation of number densities:

- For one atmosphere $1\ \text{atm} = 1{,}01 \cdot 0^{5}\ \text{Pa}$ and $T = 273\ \text{K}$ we recover the Loschmidt number n_L

$$n[\text{m}^{-3}] = \frac{1}{k_B} \times \frac{p}{T} = 7{,}25 \cdot 10^{22} \times \frac{p[\text{Pa}]}{T[\text{K}]}$$

$$n_L = 7{,}25 \cdot 10^{22} \times \frac{1{,}01 \cdot 10^{5}}{2{,}73 \cdot 10^{2}} = 2{,}66 \cdot 10^{25}\ \text{m}^{-3} = 2{,}66 \cdot 10^{19}\ \text{cm}^{-3}$$

We have $1\ \text{Pa} = 7{,}5 \cdot 10^{-3}\ \text{Torr}$ and therefore,

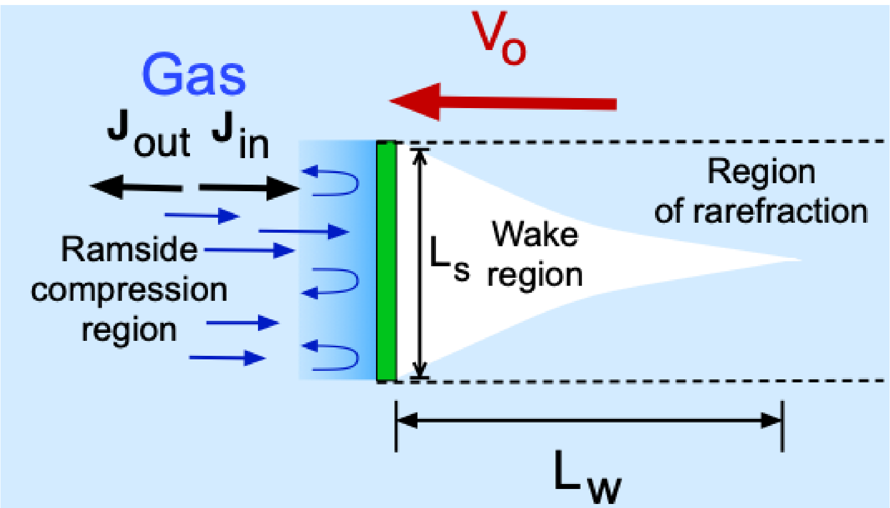
$$n[\text{m}^{-3}] = \frac{1}{k_B} \times \frac{p}{T} = 7{,}25 \cdot 10^{22} \times \frac{1}{7{,}5 \cdot 10^{-3}} \times \frac{p[\text{Torr}]}{T[\text{K}]} = 9{,}67 \cdot 10^{25}\ \frac{p[\text{Torr}]}{T[\text{K}]}$$

$1\ \text{Pa} = 10^{-2}\ \text{mbar}$

$$n[\text{m}^{-3}] = \frac{1}{k_B} \times \frac{p}{T} = 7{,}25 \cdot 10^{22} \times \frac{1}{10^{-2}} \times \frac{p[\text{mbar}]}{T[\text{K}]} = 7{,}26 \cdot 10^{24}\ \frac{p[\text{mbar}]}{T[\text{K}]}$$

Conversion to particles per cubic centimeter multiply by a 10^{-6} factor.

Neutral wake. Qualitative analysis



- A flat panel moves in LEO with orbital velocity is $V_o \gg (8k_BT/\pi\, m_o)^{1/2}$ in a region where n_o is the unperturbed neutral gas density.
- The flux of incoming particles over the ram side is $J_{in} \sim n_o(V_o - \langle v_x \rangle)$ and assuming specular inelastic reflections $J_{out} \sim n_r\,(V_o + \langle v_x \rangle)$ is the flux of outgoing molecules.
- Both are equal as $\boldsymbol{J}_{in} + \boldsymbol{J}_{out} = 0$ and thus,

  $n_r = n_o \frac{V_o - \langle V_x \rangle}{V_o + \langle V_x \rangle}$ and in front of the plate, $n = n_o + n_r = n_o \frac{2V_o}{V_o + \langle V_x \rangle} > n_o$

  Is the increased density at the ram side produces a compression region.

- In the frame that moves with the panel, the horizontal velocity of molecules filling the wake is,

$$\boldsymbol{v} = v_x\,\boldsymbol{i} + v_y\,\boldsymbol{j} = V_o\,\boldsymbol{i} - \langle v_y \rangle\,\boldsymbol{j} \quad \text{where, } \langle v_y \rangle = \left(\frac{k_B\,T}{2\pi\,m_o}\right)^{1/2} = \frac{1}{4}\left(\frac{8\,k_BT}{\pi\,m_o}\right)^{1/2} = \frac{1}{4}\bar{v}$$

and the Δt time to cover the distance $^{L_s}/_2$ is $\Delta t = {}^{2\,L_s}/_{\bar{v}}$ and in this interval the horizontal distance is,

$L_w = V_o\,\Delta t = {}^{2L_sV_o}/_{\bar{v}}$ that can be used as an estimation of the wake length past the flat panel.

- For $L_s = 1$ m, the $V_o = 7$ km/s characteristic LEO orbital velocity and $\bar{v} = 414$ m/s for N_2 at $T = 1100$ K the estimated length of the wake L_w is,

$$L_w = \frac{2\,\times\,1\text{ m}\,\times\,7.0\cdot 10^3\text{ m/s}}{414\text{ m/s}} = 33.8\text{ m}$$

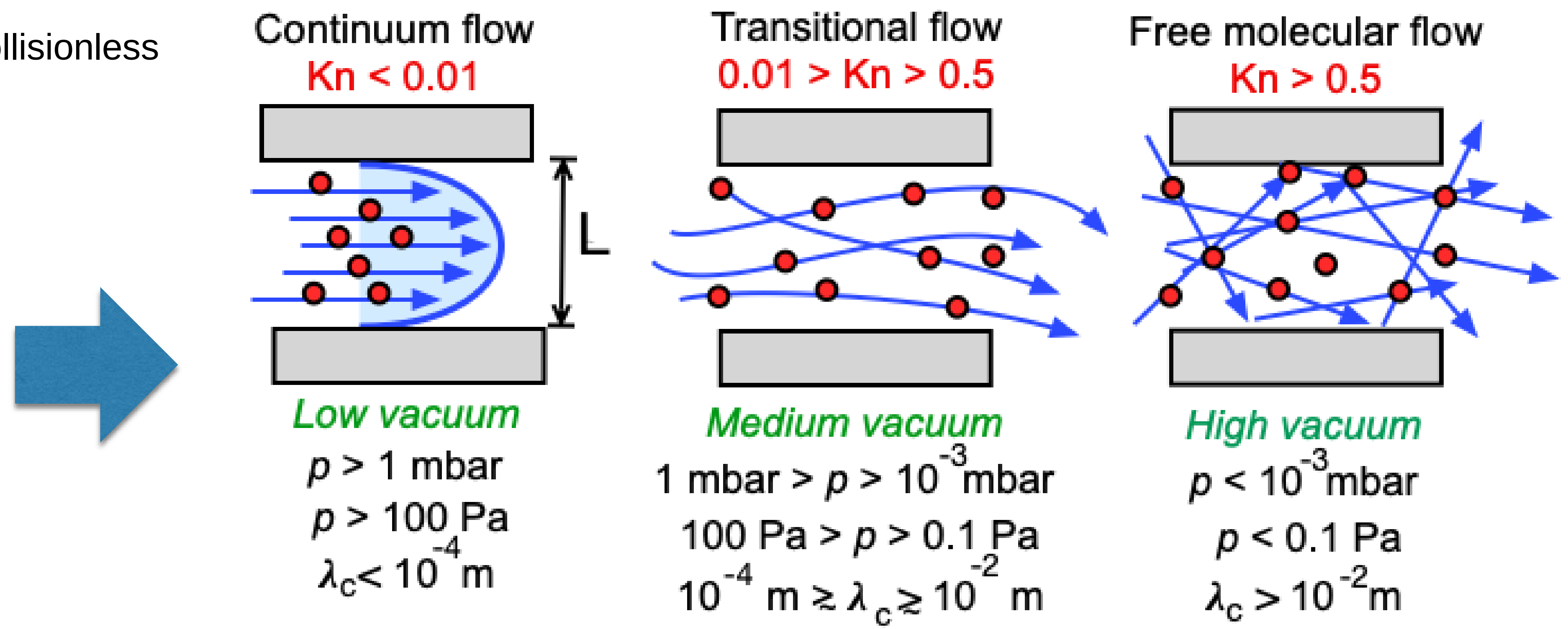
The Knudsen number

$$Kn = \frac{\text{Collisional mean free path}}{\text{Characteristic length scale}} \qquad Kn = \frac{\lambda_c}{L}$$

- $Kn \ll 1$: Collisions are dominant, and the transport phenomena are described by continuum (fluid) models.
- $Kn \gg 1$: Motion at molecular level is important and transport is described by probabilistic (kinetic) models.

- $Kn \leq 0.1$ the continuum flow around the body.
- $Kn \sim 0.5$ transitional flow dominated by inertia at the body scale.
- $Kn > 0.5$ flow around the body is collisionless and molecular motion is "ballistic".

Values of reference for characteristic pressure ranges and the corresponding values for typical mfps.



Numerical example

The interaction of ISS with ionospheric plasma is an example of practical applications of basic kinetic theory. This spacecraft **completes one orbit in 90 minutes** at the height of **400 km.** Its typical orbital velocity is therefore,

$$V_o = \frac{2\pi R}{T} = \frac{2 \times 3.14 \times (400 + 6371) \cdot 10^3}{90 \times 60} = 7.88 \text{ km/s}$$

This figure can be compared with the speed of the ion of $m_i = 28$ AMU mass (N_2). The average kinetic temperature of the ionosphere is $T_i = 0.1$ eV $= 1.160$ K $(*)$ and then,

$$\bar{v}_i = \left(\frac{8k_BT}{\pi\, m_i}\right)^{1/2} = \left(\frac{8\times1.38\cdot10^{-23}\times0.1\times11600}{3.14\times28\times1.66\cdot10^{-27}}\right)^{1/2} = 0.91 \text{ km/s} \qquad V_o \gg \bar{v}_i$$

Consequently, the ISS moves at supersonic speed with respect to the ambient plasma. This plasma flow around the ISS of ion density $n_i \simeq n_e = 5 \cdot 10^6 \text{ cm}^{-3} = 5 \cdot 10^{12} \text{ m}^{-3}$ implies an electric current density of,

$$J_e = e\, n_e V_o = 1.6 \cdot 10^{-19} \times 5 \cdot 10^{12} \times 7.88 \cdot 10^3 = 6.3 \text{ mA/m}^2$$

The ISS solar panels have a surface of $S = 2500 \text{ m}^2$ so they collect from the plasma the electric current of

$$I = J_e \times S = 6.3 \cdot 10^{-3} \times 2.5 \cdot 10^3 = 15.8 \text{ A}$$

that needs to be returned to space. Thus, spacecraft charging resulting from plasma interaction with satellites can be an issue.

(*) Data from table 1.1 in The aerodynamics of bodies in a rarefied ionized gas with applications to spacecraft environment dynamics. N. Stone. NASA technical paper 1933 (1981).

Examples. Calculation of number densities:

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Conversion to particles per cubic centimeter multiply by a 10^{-6} factor.

The Maxwellian equilibrium plasma

- A Maxwellian plasma is in thermal equilibrium where all particle groups have the same kinetic temperature

$$k_B T_e = k_B T_i = k_B T$$

and $k_B T$ is the uniform temperature corresponding to the thermodynamic equilibrium.

- Plasma quasineutrality, property $n_i \cong n_e$ implies that no electric fields exists in the equilibrium plasma bulk; uniform plasma potential.

$\nabla \cdot E = \frac{e}{\varepsilon_o}(n_i - n_e) \simeq 0$ then, $E = -\nabla\phi \simeq 0$ and $\phi(r) \simeq \phi_o$ is uniform in space.

- In the thermal equilibrium there is no transport; no macroscopic particle currents and no gradient of particle densities.

$$\nabla n_e = \nabla n_i = 0$$

- Under an externally applied electric potential $\phi(x)$ we have for ions and electrons ,

$$E_\alpha = \frac{m_\alpha \boldsymbol{v}_\alpha^2}{2} \pm e\phi(x) = E_\alpha \pm e\phi(x)$$

and integration $n_\alpha = n_{\alpha o} \int_{-\infty}^{+\infty} f_\alpha(\boldsymbol{v}_\alpha)\, d^3 v_\alpha = n_{\alpha o} \int_0^{\infty} g_\alpha(E_\alpha)\, dE_\alpha$ gives,

$$n_e = n_{eo} \exp\left(\frac{e\phi(x)}{k_B T}\right) \quad \text{and} \quad n_i = n_{io} \exp\left(-\frac{e\phi(x)}{k_B T}\right)$$

Length and time fluctuations

The damping and/or attenuation of *small amplitude fluctuations* of the equilibrium state takes place over characteristic length and time scales.

- Space fluctuations: Debye length
- Time fluctuations: Plasma frequency
- Number of charges: Plasma parameter

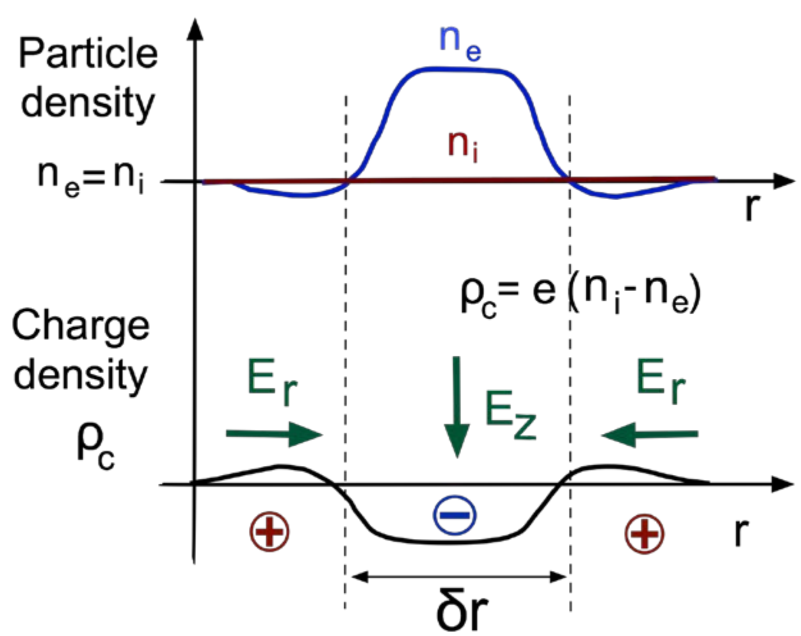
A localized perturbation $\boldsymbol{E}_1$ of the $\boldsymbol{E}_o \sim 0$ equilibrium electric field is produced by small changes in the unperturbed $n_{eo} \simeq n_{io}$ electron and/or ion densities.

$$\left\{ \begin{array}{c} n_i\,(\boldsymbol{r}) \neq n_e(\boldsymbol{r}) \\ \boldsymbol{E} \simeq \underset{\searrow 0}{\boldsymbol{E}_o} + \boldsymbol{E}_1 \end{array} \right\} \qquad \nabla \cdot \boldsymbol{E}_1 = \frac{e}{\varepsilon_o}(n_i - n_e)$$

The *Debye lengths* for ions λ_{Di} and electrons λ_{De} with temperatures ($k_B T_i$, $k_B T_e$) are the characteristic longitudes for the equilibrium $n_{eo} \simeq n_{io}$ plasma

$$\lambda_{De} = \sqrt{\frac{\varepsilon_o k_B T_e}{e^2\, n_{eo}}} \qquad \lambda_{Di} = \sqrt{\frac{\varepsilon_o k_B T_e}{e^2\, n_{io}}}$$

Usually $k_B T_e \gg k_B T_i$ and then $\lambda_{De} \gg \lambda_{Di}$



There is two time scales associated to the *plasma frequencies* of ions and electrons, that only depend on the particle densities.

$$\omega_{pe} = \sqrt{\frac{e^2 n_{eo}}{\varepsilon_o m_e}} \qquad \omega_{pi} = \sqrt{\frac{e^2 n_{io}}{\varepsilon_o m_i}}$$

$$\omega_{pi} \ll \omega_{pe} = \omega_{pi}\sqrt{\frac{m_i}{m_e}}$$

Debye shielding

- In the initial plasma equilibrium, we introduce[(*)] a small perturbation $\delta\rho_c = q\,\delta(\boldsymbol{r})$ in the electric charge

$$\left.\begin{aligned}\delta\rho_{ext} &= q\,\delta(\boldsymbol{r}) \\ \delta\rho_{sp}(\mathrm{r}) &= e\,[n_i(\boldsymbol{r}) - n_e(\boldsymbol{r})]\end{aligned}\right\} \quad \left.\begin{aligned}&\boldsymbol{E}(\boldsymbol{r}) \simeq \underbrace{\boldsymbol{E}_o}_{0} + \boldsymbol{E}_1(\boldsymbol{r}) \\ &\boldsymbol{E}_o \simeq 0 \text{ (plasma equilibrium)}\end{aligned}\right\} \quad \boldsymbol{E}_1 = -\nabla\,\varphi_1(\boldsymbol{r})$$

$\boldsymbol{E}_1(\boldsymbol{r})$ is the perturbed electric field governed by the Poisson equation

$$\nabla\cdot\boldsymbol{E}_1 = -\nabla^2\varphi_1 = \frac{\delta\rho_c + \delta\rho_{sp}}{\varepsilon_o} \qquad \nabla\cdot\boldsymbol{E}_1 = \frac{q}{\varepsilon_o}\delta(\boldsymbol{r}) + \frac{e}{\varepsilon_o}[n_{i1}(\boldsymbol{r}) - n_{e1}(\boldsymbol{r})]$$

- Charge $(\alpha = e, i)$ density fluctuations $n_e(\boldsymbol{r})$ and $n_i(\boldsymbol{r})$ in a Maxwellian plasma in equilibrium $(n_{eo} \simeq n_{io} = n_o)$ are given by,

$$n_\alpha(r) = n_o \exp\left(\pm\frac{e\,\varphi_1(\boldsymbol{r})}{k_B T_\alpha}\right)$$

- Small amplitude perturbations of the charge/electric field means that thermal energy $k_B T$ dominates over the electrostatic energy $|e\varphi_1(\boldsymbol{r})|$ in this case we said this is an ideal plasma then we can approximate for ions and electrons

$$\left|\frac{e\,\varphi_1(\boldsymbol{r})}{k_B T_\alpha}\right| \ll 1 \qquad n_\alpha(r) \simeq n_o\left(1 \pm \frac{e\,\varphi_1(\boldsymbol{r})}{k_B T_\alpha}\right)$$

$$-\nabla^2\varphi_1(\boldsymbol{r}) = \frac{1}{\varepsilon_o}\left[\delta\rho_{ext} + \frac{e^2\,n_o}{k_B T_i}\varphi_1(\boldsymbol{r}) + \frac{e^2\,n_o}{k_B T_e}\varphi_1(\boldsymbol{r})\right]$$

(*) Details of this derivation in the note entitled The ideal Maxwellian plasma available in the supplementary material to these these lectures.

Plasma time scale: the plasma frequency

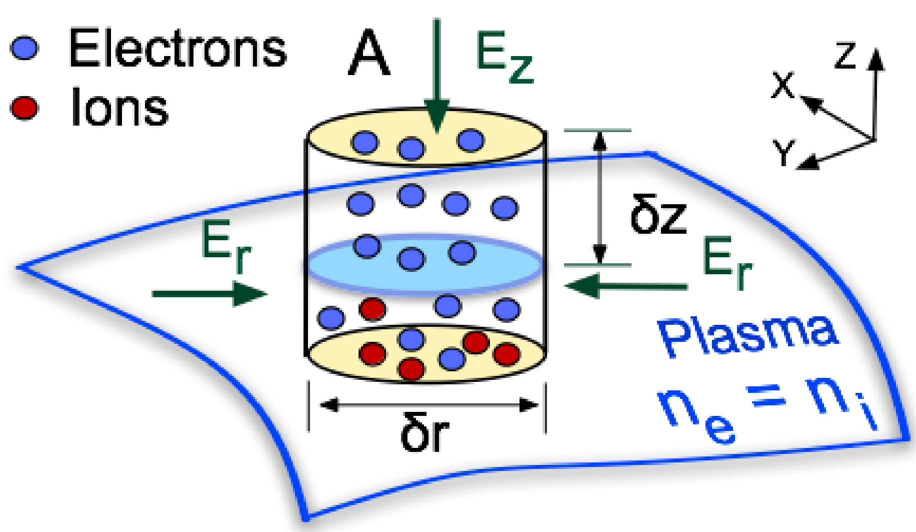
We start again with the equilibrium conditions $\boldsymbol{E} \simeq 0$ and densities $n_{eo} \simeq n_{io} = n_o$

Few $N_e = n_o\, \delta V$ electrons (blue dots) scape from the plasma inside the upper side of the pillbox(*) of section A and $\delta Q = -e\, n_o\, \delta V$ is the negative charge inside with mass $M_e = N_e m_e$

$$\delta Q = -e\, n_o\, \delta V = -e\, n_o (A\, \delta z)$$

$$\frac{\delta Q}{\varepsilon_o} = \int_{box} \boldsymbol{E}_1 \cdot d\boldsymbol{s} \rightarrow E_1 = \frac{e}{\varepsilon_o}\, n_o\, \delta z$$

$$\left.\begin{aligned} & M_e \frac{dV_{cm}}{dt} = \delta Q\, \boldsymbol{E}_1 \quad \frac{dV_{cm}}{dt} = \frac{d^2}{dt^2}(\delta z) \\ & N_e = n_o\, \delta V = n_o\, A\delta z \\ & \frac{d^2 N_e}{dt^2} + \left(\frac{e^2\, n_o}{m_e \varepsilon_o}\right) N_e = 0 \end{aligned}\right\} \quad \omega_{pe} = \sqrt{\frac{e^2\, n_o}{m_e\, \varepsilon_o}}$$



This equation for the oscillation of N_e electrons and defines the electron plasma frequency. Similar arguments apply for ions, and we have for $\alpha = e, i$

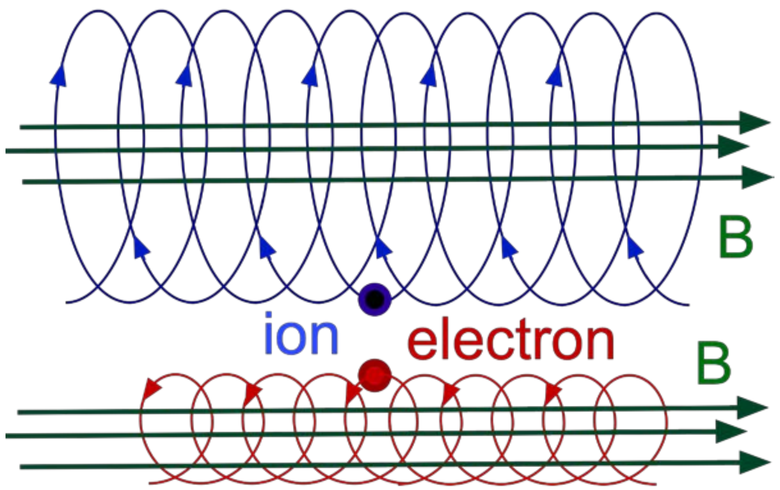
$$\omega_{p\alpha} = \sqrt{\frac{e^2\, n_o}{m_\alpha\, \varepsilon_o}}$$

Both frequencies depend on the equilibrium plasma density n_o

and always, $$\frac{\omega_{pi}}{\omega_{pe}} = \sqrt{\frac{m_e}{m_i}}$$

(*) Details of this derivation in the note entitled The ideal Maxwellian plasma available in the supplementary material these these lectures.

Larmor radius in magnetized plasmas



- The non-relativistic force experienced by the charge q_α where $\alpha = e, i$ is,

$$\boldsymbol{F}_\alpha = q_\alpha n_\alpha \left(\boldsymbol{E} + \boldsymbol{v}_\alpha \wedge \boldsymbol{B}\right)$$

- The charged particle q_α makes a circular motion around the magnetic field lines with the cyclotron frequency Ω_α Where $B_\perp$ is the perpendicular component to magnetic field line,

$$\Omega_\alpha = \frac{q_\alpha B_\perp}{m_\alpha}$$

- This introduces a new characteristic length, the gyroradius of Larmor radius is the ratio between the perpendicular component of the velocity and the gyrofrequency

$$r_{L\alpha} = \frac{m_\alpha v_\perp}{|q_\alpha| B}$$

$$v_{th,e} = \sqrt{\frac{2 k_B T_e}{m_e}} \qquad R_{Le} = \frac{v_{th,e}}{\Omega_e} \qquad R_{Le} = R_{Li}\sqrt{\frac{m_e}{m_i}\frac{T_e}{T_i}}$$

$$v_{th,i} = \sqrt{\frac{2 k_B T_i}{m_i}} \qquad R_{Li} = \frac{v_{th,i}}{\Omega_i} \qquad R_{Le} \ll R_{Li}$$

Magnetized electrons and unmagnetized ions $\quad R_{Le} \ll R_{Li} > L$

Magnetized electrons and ions $\quad R_{Le} \ll R_{Li} < L$

Macroscopic transport equations (plasma fluid)

Transport equations for neutral gases: The medium is considered as a continuum on a macroscopic length scale L_s.

$$K_n = \frac{\lambda_c}{L_s} \ll 1$$

- $f(\boldsymbol{r}, \boldsymbol{v}, t)$ changes smoothly over (L_s, T_s)
- Averages give the macroscopic physical magnitudes

- Fluid velocity $\boldsymbol{u}(\boldsymbol{r}, t)$
- Temperature $T(\boldsymbol{r}, t)$
- Particle density (pressure) $n(\boldsymbol{r}, \mathrm{t}) \leftrightarrow p(\boldsymbol{r}, t)$

These macroscopic physical magnitudes are calculated as statistical averages over a length scale ℓ such as $L_s \gg \ell \gg \lambda_c$ and within the volume $\delta V \sim \ell^3$ the number of particles $N = \ell^3 n$ is huge.

Transport equations for plasmas: A plasma can be considered as a macroscopic continuum medium under certain conditions.

- Ions, electrons and neutral atoms are considered as *three mutually interpenetrating* fluids whose motions are coupled by short-range collisions and long-range electromagnetic interactions.
- The physical magnitudes $(\alpha = e, i, a)$ are *macroscopic*: $\boldsymbol{u}_\alpha(\boldsymbol{r}, t)$, $n_\alpha(\boldsymbol{r}, t)$, $T_\alpha(\boldsymbol{r}, t)$.
- Electric field *shielding requires of countless charges* $N_\alpha = n_\alpha(\boldsymbol{r}, t)\, \lambda_D^3$ within the Debye length scale.

Within a length scale $\ell \ll L_s$ small compared with the dimensions of the system.

- $L_s \gg \ell \gtrsim \lambda_D$
- $L_s \gg \ell \gg \lambda_c$
- $N_\alpha = n_\alpha(\boldsymbol{r}, t)\, \ell^3$ is huge

In these conditions, collisions relax small random fluctuations of energy $\delta E_\alpha \sim \delta T_\alpha \ll T_\alpha$ and densities $\delta n_\alpha \ll n_\alpha$ over a time scale much shorter than the macroscopic time $\tau \ll T_s \sim L_s/|\boldsymbol{u}_\alpha|$ scale associated to plasma motion (LTE plasmas)

Approximated magnitudes in typical plasmas

Plasma	n_e (cm^{-3})	$k_B T_e$ (eV)	λ_D(cm)	f_{pe} (Hz)	$n_e\, \lambda_{De}^3$
Interstellar gas	1	1	700	6.0×10^4	4×10^8
Solar corona	10^9	100	0,2	2.0×10^9	8×10^6
Solar atmosphere Gas discharge	10^{14}	1	7.0×10^{-5}	6.0×10^{11}	40
Tokamak	10^{14}	10^4	2.0×10^{-3}	2.0×10^{12}	6×10^6

Values from NRL Plasma Formulary 2019. This practical reference could be downloaded for free at;
https://www.nrl.navy.mil/ppd/content/nrl-plasma-formulary

Application: Propagation electromagnetic waves in a plasma

We start with two Maxwell equations in vacuum to derive the dispersion equation $\omega(\boldsymbol{k})$ for the propagation of transversal electromagnetic (EM) waves in a plasma.

$$\left.\begin{aligned} \nabla \wedge \boldsymbol{B} &= \mu_o\, \boldsymbol{J}_T + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} \\ \nabla \wedge \boldsymbol{E} &= -\frac{\partial \boldsymbol{B}}{\partial t} \end{aligned}\right\}$$

$$-\nabla \wedge (\boldsymbol{\nabla} \wedge \boldsymbol{E}) = -\nabla(\nabla\cdot\boldsymbol{E}) + \nabla^2\boldsymbol{E} = \mu_o \frac{\partial \boldsymbol{J}_T}{\partial t} + \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$

Assuming the usual dependence as $\exp[i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega\, t)]$ for all variables with the changes,

$$\nabla\phi \to i\boldsymbol{k}\,\phi; \quad \nabla\cdot\boldsymbol{E} \to \boldsymbol{k}\cdot\boldsymbol{E}; \quad \nabla\wedge\boldsymbol{E} \to \boldsymbol{k}\wedge\boldsymbol{E}; \quad \nabla^2\boldsymbol{E} \to (\boldsymbol{k}\cdot\boldsymbol{k})\,\boldsymbol{E}\,; \quad {}^{\partial}/_{\partial t} \to (-i\omega);$$

As for planar waves $\boldsymbol{k} \perp \boldsymbol{E}$

We have,

$$-\nabla(\underbrace{\boldsymbol{k}\cdot\boldsymbol{E}}_{0}) + (\boldsymbol{k}\cdot\boldsymbol{k})\boldsymbol{E} = \mu_o\,(-i\,\omega)\,\boldsymbol{J}_T + \frac{1}{c^2}\,(-i\,\omega)^2\,\boldsymbol{E}$$

and this equation reads,

$$\left(\frac{\omega^2}{c^2} - k^2\right)\boldsymbol{E} = \mu_o\,(-i\,\omega)\,\boldsymbol{J}_T$$

These approximations imply a constitutive equation for the plasma

- The charge transport current $\boldsymbol{J}_T \simeq -e\,n_o\boldsymbol{u}_e$ essentially due to the electrons (faster time scale).
- Charge transport $\boldsymbol{J}_T = \sigma_p\,(\omega)\boldsymbol{E}$ is given by the Ohm law, where $\sigma_p(\omega)$ is the plasma conductivity.

We need of a simple model for the plasma electron conductivity $\sigma_p(\omega)$ for closure.

$$m_e \frac{\partial \boldsymbol{u}_e}{\partial t} = -e\,\boldsymbol{E} \qquad m_e\,(-i\omega)\,\boldsymbol{u}_e = -e\,\boldsymbol{E}$$

$$\sigma_p(\omega) \simeq \frac{e^2\,n_o}{m_e\,(i\omega)} = -i\,\varepsilon_o\,\frac{\omega_{pe}^2}{\omega}$$

Finally,

$$\omega^2 = \omega_{pe}^2 + k^2c^2$$

where,

$$k = \frac{2\pi}{\lambda}$$

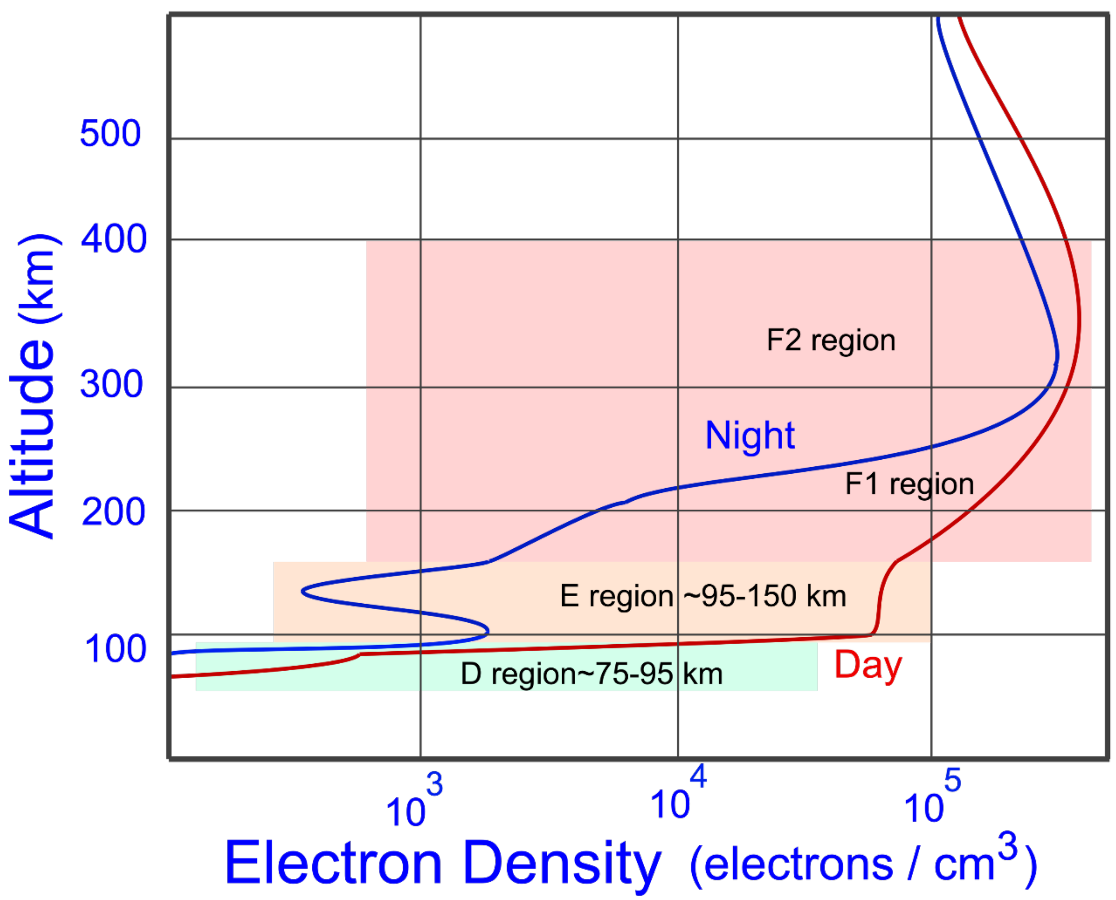
EM wave transmission in the ionosphere

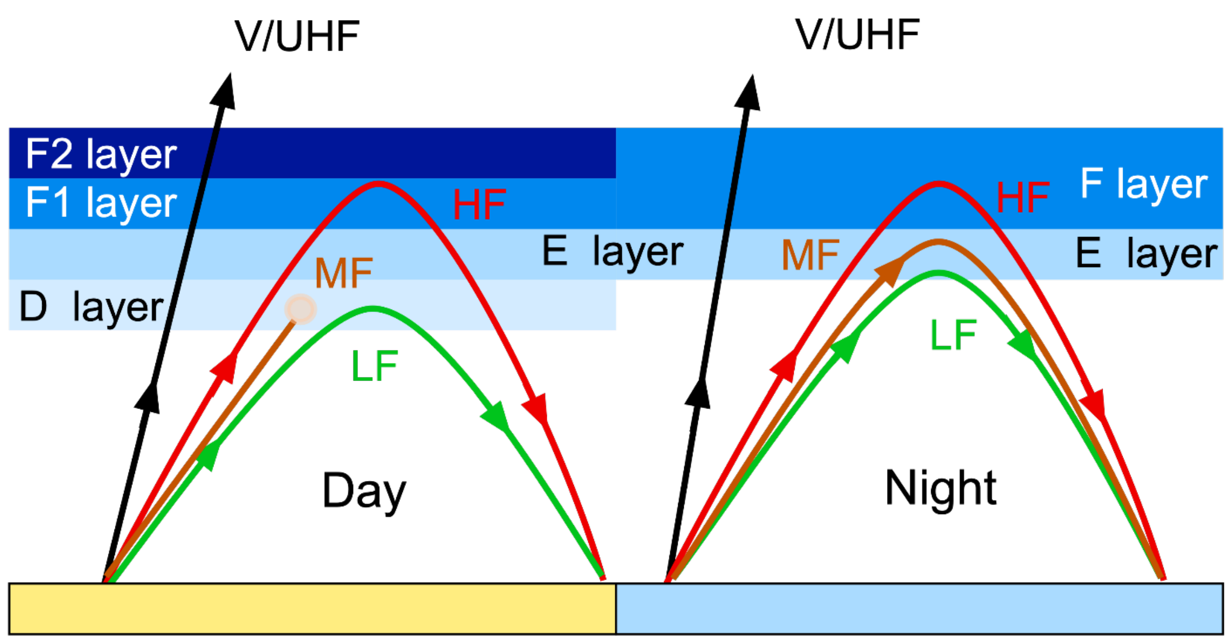
$$\omega^2 = \omega_{pe}^2 + k^2c^2$$

$$k^2 = \frac{\omega_{pe}^2}{c^2}\left(\frac{\omega^2}{\omega_{pe}^2} - 1\right)$$

- When the EM wave frequency $\omega < \omega_{pe}$ the wave number $k = 2\pi/\lambda$ becomes a complex number. As the plasma is an electrically conductive medium, the EM waves are reflected and/or can be absorbed in this case.
- The plasma density determines the *minimum EM wave frequency* that can be transmitted across an ionospheric layer of a given plasma density.

$$\omega_{pe} = \sqrt{\frac{e^2\, n_o}{m_e\, \varepsilon_o}}$$





Plasmas at main Earth orbits

The plasma environment affects spacecrafts in any orbit that can be roughly divided in:

- LEO (Low Earth Orbit): the plasma is cold and dense ($n_e \sim 10^{10} - 10^{11}\ \text{part./m}^3$) in the plasmasphere (altitudes below 2000 km) and can be approximated by a Maxwellian energy distribution.
- PEO (Polar Earth Orbit): in addition to dense plasma at low altitudes exist fluxes of energetic particles from the solar activity transported by the geomagnetic field.
- GEO (Geostationary Orbit): the plasma density drops ($n_e \sim 10^{8} - 10^{9}\ \text{part./m}^3$) and energy distributions are not Maxwellian with higher mean energy.

- Spacecrafts (typical dimension L_s) interact with plasmas (natural and/or artificial, created by the vehicle itself) in many ways and in LEO $\lambda_D < L_S$ whereas $\lambda_D > L_S$ in GEO orbits.
- Dielectric and metallic component are immersed in an electrically active medium where orbital speed also induces potential differences.
- For the typical orbital velocity $v_o \sim 7\ \text{km}/s$ in LEO the induced electric field is $\boldsymbol{E} = \boldsymbol{v}_o \times \boldsymbol{B}$ we can estimate, $E \sim v_o B$

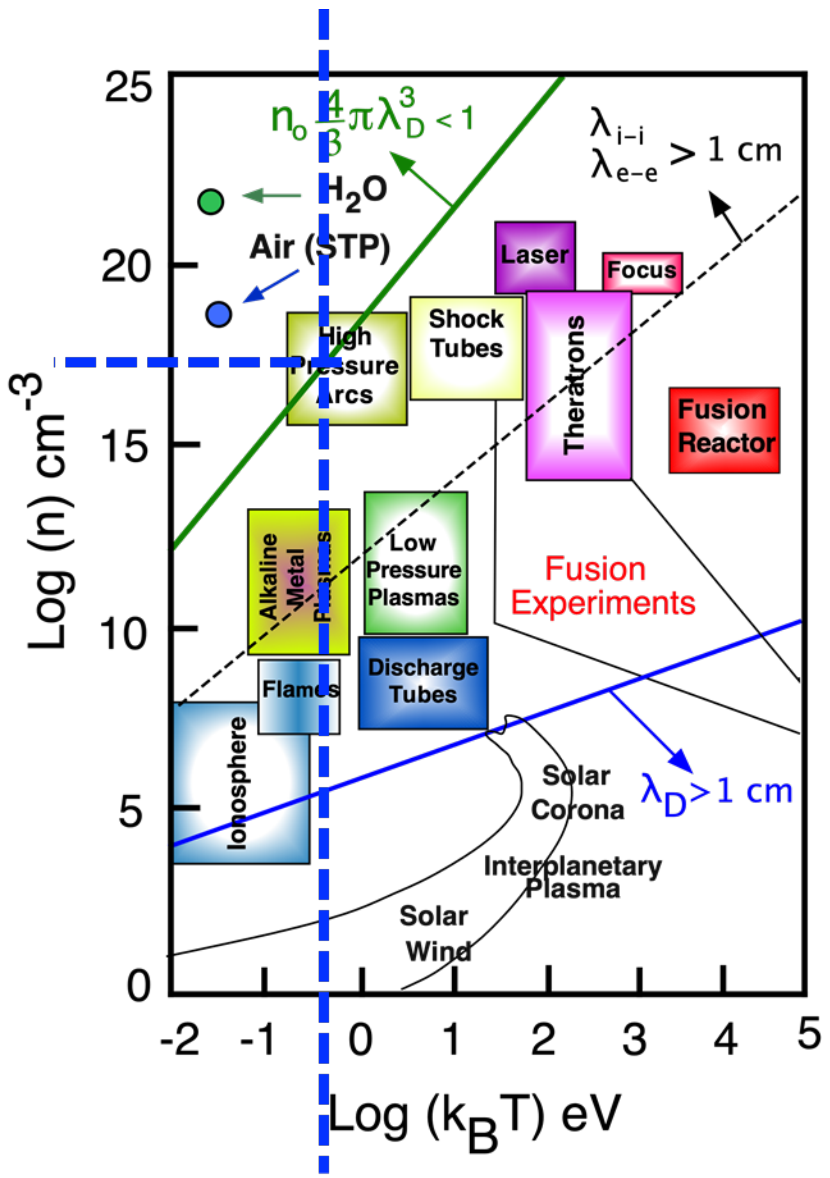
EOS (700 km)	$B = 0.14 \cdot 10^{-4}\ \text{T}$	$E \sim 9.8 \cdot 10^{-2}\ \text{V}/\text{m}$
ISS (500 km)	$B = 0.23 \cdot 10^{-4}\ \text{T}$	$E \sim 16.0 \cdot 10^{-2}\ \text{V}/\text{m}$

- Along the 109 m length of the ISS solar array, we have a voltage drop of:

$$\Delta V = 16 \cdot 10^{-2} \times 1.09 \cdot 10^{2} = 17.4\ V$$

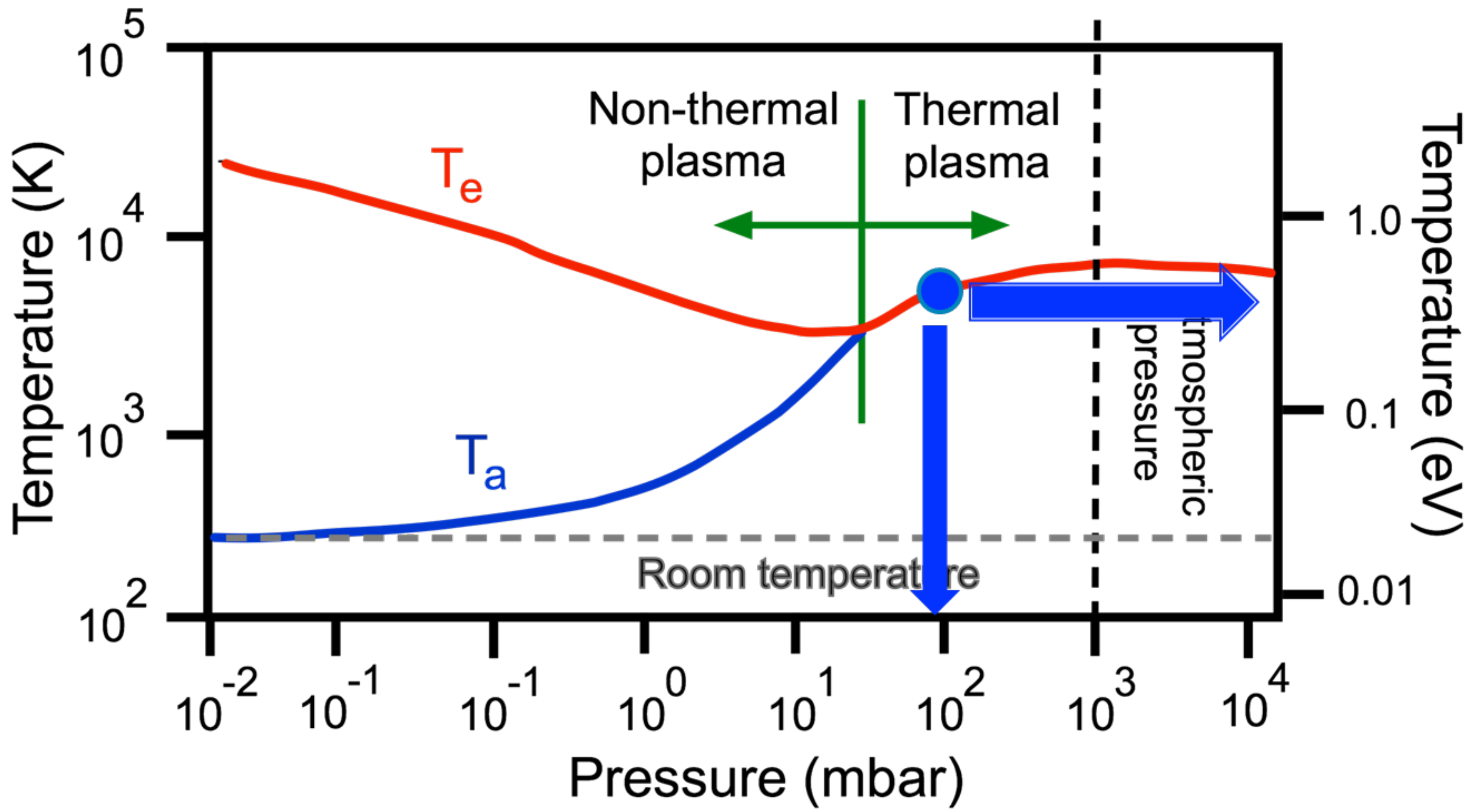
this medium is thus electrically active.

Some LTE plasma calculations



$n_e \sim 10^{16}\ \mathrm{cm}^{-3}$

$T_e \sim 0.7$ eV



p~ 100 mbar

Thermal plasma (data from figures)

$$\left\{\begin{array}{l} T_i \simeq T_a \simeq T_e = 0.7\ \mathrm{eV} \to T_a = 8120\ \mathrm{K} \\ p \simeq 100\ \mathrm{mbar} \to n_a = 8.9 \cdot 10^{16}\ \mathrm{cm}^{-3} \\ n_e = 10^{16}\ \mathrm{cm}^{-3} \\ \text{Argon gas}\ \ m_a = 40\ \mathrm{amu} \end{array}\right\}$$

$$\sigma_{ea} = 5.0 \cdot\ 10^{-15}\ \mathrm{cm}^2$$

$$\sigma_{ia} = 6.0 \cdot\ 10^{-15}\ \mathrm{cm}^2$$

$$\lambda_D = 6.22 \cdot 10^{-6}\ \mathrm{cm}$$

- Ionization degree: $\alpha = \frac{n_e}{n_a} = \frac{10^{16}}{8.9 \cdot\ 10^{16}} = 0.112$ or equivalently, $\alpha \sim 11\ \%$
- Collisional mean free path: $\lambda_c = \frac{1}{n_a\, \sigma_{ea}} = \frac{1}{8.9 \cdot\ 10^{16} \times 5.0 \cdot 10^{-15}} = \frac{1}{89 \times 5} \to \lambda_c = 2.25 \cdot 10^{-3}\ \mathrm{cm}$

$$\left\{\begin{array}{l} N_a = n_a \times \lambda_c^3 = 8.9 \cdot\ 10^{16} \times (2.25 \cdot\ 10^{-3})^3 = 8.9 \cdot 10^{16} \times 11.24 \cdot 10^{-9} \to N_a = 10^9\ \text{particles} \\ N_e = n_e \times \lambda_c^3 = 10^{16} \times (2.25 \cdot 10^{-3})^3 = 10^{16} \times 11.39 \cdot 10^{-9} \to N_e = 1.14 \cdot 10^8\ \text{charges} \end{array}\right.$$

- Plasma parameter:

$$N_D = n_e \times \lambda_D^3 = 10^{16} \times (6.22 \cdot 10^{-6})^3 = 11.24 \cdot 10^{16} \times 2.41 \cdot 10^2 \cdot 10^{-18} \to N_D = 271\ \text{charges}$$

- Mean thermal velocities:

$$V_{th,e} = \sqrt{8k_B T_e/\pi\, m_e} = 5.6 \cdot 10^5\ \mathrm{m/s} = 5.6 \cdot 10^7\ \mathrm{cm/s}$$

$$V_{th,i} = \sqrt{8k_B T_i/\pi\, m_i} = 2.1 \cdot 10^3\ \mathrm{m/s} = 2.1\ \mathrm{km/s}$$

- Collision frequencies:

$$n_e = n_i$$

$$\nu_{ea} = \sigma_{ea}\, n_e\, n_a\, V_{th,e} = (5\cdot 10^{-15}) \times (10^{10}) \times (8.9\cdot 10^{22}) \times (5.6\cdot 10^{7}) = 2.5\cdot 10^{26} \text{ cols.}/(\text{cm}^3 \times \text{s})$$

$$\nu_{ia} = \sigma_{ia}\, n_i\, n_a\, V_{th,i} = (6\cdot 10^{-15}) \times (10^{10}) \times (8.9\cdot 10^{22}) \times (2.1\cdot 10^{3}) = 1.12\cdot 10^{22} \text{ cols.}/(\text{cm}^3 \times \text{s})$$

- Plasma frequencies:

$$n_e = n_i = n_o \quad \begin{cases} f_{pe} = \dfrac{\omega_{pe}}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{n_o\, e^2}{\varepsilon_o\, m_e}} = 8.98\cdot 10^{8}\text{ Hz} = 8.98\cdot 10^{2}\text{ MHz} \\ f_{pi} = \dfrac{\omega_{pi}}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{n_o\, e^2}{\varepsilon_o\, m_i}} = 3.31\cdot 10^{6}\text{ Hz} = 3.31\text{ MHz} \end{cases}$$

- For the characteristic distance: $\ell \simeq 10^{-2}\text{ cm} \gg \lambda_c \gg \lambda_D$

$$\begin{cases} N_a = n_a \times \ell^3 = (8.9\cdot 10^{16}) \times (10^{-2})^3 = 8.9\cdot 10^{16} \times 10^{-6} \rightarrow N_a = 8.9 \times 10^{10} \text{ particles} \\ N_e = n_e \times \ell^3 = (10^{16}) \times (10^{-2})^3 = 10^{16} \times 10^{-6} \rightarrow N_e = 10^{10} \text{ charges} \end{cases}$$

Ionosphere
Altitude ~ 300 km

$$\left\{\begin{array}{l} T_i \simeq T_a \simeq T_e = 0.24\ \text{eV} \rightarrow T_a = 2784\ \text{K} \\ p \simeq 5 \cdot 10^{-5}\ \text{Pa} = 5 \cdot 10^{-7}\,\text{mbar} \\ n_a = 1.3 \cdot 10^{\,9}\ \text{cm}^{-3} \\ n_e = 5 \cdot 10^{\,5}\ \text{cm}^{-3}\ \text{(F-layer)} \\ \text{Oxygen}\ \ m_a = 16\ \text{amu} \end{array}\right\}$$

$\sigma_{ea} = 5.0 \cdot 10^{-15}\ \text{cm}^2$

$\sigma_{ia} = 3.0 \cdot 10^{-15}\ \text{cm}^2$

$\lambda_D = 5.2 \cdot 10^{-3}\ \text{m} = 0.52\ \text{cm}$

- Ionization degree: $\alpha = \dfrac{n_e}{n_a} = \dfrac{5 \cdot 10^5}{1.3 \cdot 10^9} = 3.85 \cdot 10^{-4}$ $\quad \alpha \sim {}^{4}/_{10.000}$
- Collisional mean free path:

$$\lambda_c = \frac{1}{n_a\,\sigma_{ea}} = \frac{1}{1.3 \cdot 10^9\ \times\ 5.0 \cdot 10^{-15}} = \frac{10^4}{1.3\ \times\ 5} \rightarrow \lambda_c = 1.54 \cdot 10^5\ \text{cm} = 1.54\ \text{km}$$

$$\left\{\begin{array}{l} N_a = n_a \times \lambda_c^3 = 1.3 \cdot\ 10^{\,9}\ \times\ (1.54 \cdot\ 10^5)^{\,3} = 1.3 \cdot 10^9\ \times\ 3.65 \cdot 10^{15} \rightarrow N_a = 4.75 \cdot 10^{\,24}\ \text{particles} \\ N_e = n_e\ \times\ \lambda_c^3 = 5 \cdot 10^{\,5}\ \times\ (1.54 \cdot 10^5)^{\,3} =\ 5 \cdot 10^5\ \times\ 3.65 \cdot 10^{15} \rightarrow N_e\ = 1.83 \cdot 10^{21}\ \text{charges} \end{array}\right.$$

- Plasma parameter: $N_D = n_e\ \times\ \lambda_D^3 = 5 \cdot 10^{\,5}\ \times\ (0.52)^3 = 0.70 \cdot 10^5 \rightarrow N_D\ = 7 \cdot 10^4$ charges

- Mean thermal velocities:

$$V_{th,e} = \sqrt{8k_B T_e/\pi\, m_e} = 3.28 \cdot 10^5 \ \text{m/s} = 3.28 \cdot 10^7 \ \text{cm/s}$$

$$V_{th,i} = \sqrt{8k_B T_i/\pi\, m_i} = 1.91 \cdot 10^3 \ \text{m/s} = 1.91 \ \text{km/s}$$

- Collision frequencies: $n_e = n_i$

$$\nu_{ea} = \sigma_{ea}\, n_e\, n_a\, V_{th,e} = (5 \cdot 10^{-15}) \times (5 \cdot 10^5) \times (1.3 \cdot 10^9) \times (3.28 \cdot 10^7) = 1.06 \cdot 10^{8} \ \text{cols.}/(\text{cm}^3 \times \text{s})$$

$$\nu_{ia} = \sigma_{ia}\, n_i\, n_a\, V_{th,i} = (3 \cdot 10^{-15}) \times (5 \cdot 10^5) \times (1.3 \cdot 10^9) \times (1.91 \cdot 10^3) = 3.72 \cdot 10^3 \ \text{cols.}/(\text{cm}^3 \times \text{s})$$

- Plasma frequencies:

$$n_e = n_i = n_o \quad \begin{cases} f_{pe} = \dfrac{\omega_{pe}}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{n_o\, e^2}{\varepsilon_o\, m_e}} = 6.35 \cdot 10^6 \ \text{Hz} = 6.35 \ \text{MHz} \\[2ex] f_{pi} = \dfrac{\omega_{pi}}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{n_o\, e^2}{\varepsilon_o\, m_i}} = 3.70 \cdot 10^4 \ \text{Hz} = 37 \ \text{kHz} \end{cases}$$

- For the characteristic distance: $\ell \simeq 2 \ \text{km} = 2 \cdot 10^3 \ \text{m} = 2 \cdot 10^5 \ \text{cm} \gg \lambda_c \gg \lambda_D$

$$\begin{cases} N_a = n_a \times \ell^3 = (1.3 \cdot 10^9) \times (2 \cdot 10^5)^3 = 10.4 \times 10^{9} \times 10^{15} \rightarrow N_a = 1.04 \times 10^{25} \ \text{particles} \\[1ex] N_e = n_e \times \ell^3 = (5 \cdot 10^5) \times (2 \cdot 10^5)^3 = 40 \times 10^5 \times 10^{15} \rightarrow N_e = 4 \cdot 10^{21} \ \text{charges} \end{cases}$$