Máster Universitario en Ingeniería Aeronáutica

The Space Environment

Hydrodynamic description of plasmas.
Momentum equations

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Momentum transport equation

- This equation of motion states the time evolution of momentum for each fluid element is due to self-consistent electromagnetic forces, pressure, shear and collisional interaction, as well as the creation/recombination of particles.

Charge and current density

$$\begin{cases} \rho_{e\alpha} = q_\alpha \, n_\alpha \\ \boldsymbol{J}_{e\alpha} = \rho_{e\alpha} \, \boldsymbol{u}_\alpha = n_{e\alpha} \, q_\alpha \, \boldsymbol{u}_\alpha \end{cases}$$

$\rho_{m\alpha} = m_\alpha \, n_\alpha$ Mass density

$p_\alpha = n_\alpha \, k_B T_\alpha$

Momentum loss/gain by ionization/recombination

$$\rho_{m\alpha}\left(\frac{\partial \boldsymbol{u}_\alpha}{\partial t} + (\boldsymbol{u}_\alpha \cdot \nabla)\, \boldsymbol{u}_\alpha\right) = -\nabla p_\alpha - \nabla \cdot \boldsymbol{\Pi}_\alpha - m_\alpha \boldsymbol{u}_\alpha (S_\alpha - L_\alpha) + (\rho_{e\alpha} \, \boldsymbol{E} + \boldsymbol{J}_{e\alpha} \wedge \boldsymbol{B}) + \boldsymbol{R}_\alpha$$

Scalar (isotropic) pressure

Non-diagonal components of the stress tensor (usually neglected)

Self-consistent electromagnetic force

Friction force

- The friction force between species accounts for the collisional interaction between electrons, ions and neutrals.

$$\boldsymbol{R}_{\alpha\alpha} = 0 \quad \boldsymbol{R}_\alpha = \sum_{\beta \neq \alpha} \boldsymbol{R}_{\alpha\beta} \quad \boldsymbol{R}_{\alpha\beta} = -\boldsymbol{R}_{\beta\alpha} \quad \sum_\alpha \boldsymbol{R}_\alpha = 0$$

$$\boldsymbol{R}_{\alpha\beta} = -\mu_{\alpha\beta} \, n_\alpha \, \nu_{\alpha\beta} (\boldsymbol{u}_\alpha - \boldsymbol{u}_\beta)$$

$$\begin{cases} \mu_{\alpha\beta} = \dfrac{m_\alpha \, m_\beta}{m_\alpha + m_\beta} \\ \mu_{ea} \simeq m_e \end{cases}$$

Where ν_{ea} is the collision frequency

The electromagnetic fields $\boldsymbol{E}(\boldsymbol{r}, t)$ and $\boldsymbol{B}(\boldsymbol{r}, t)$ are solutions of the Maxwell equations

$$\nabla \cdot \boldsymbol{E} = \frac{e}{\varepsilon_o}(n_i - n_e) \quad \nabla \wedge \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \quad \boldsymbol{J}_c = \sum_\alpha \boldsymbol{J}_{e\alpha}$$

$$\nabla \cdot \boldsymbol{B} = 0 \quad \nabla \wedge \boldsymbol{B} = \mu_o \boldsymbol{J}_c + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}$$

Charge-exchange

$$A_{slow} + {A_{fast}}^{+} \longrightarrow {A_{slow}}^{+} + A_{fast}$$

Source of **fast atoms** and **slow ions**

Ion-Thruster Plume Modeling for Backflow Contamination

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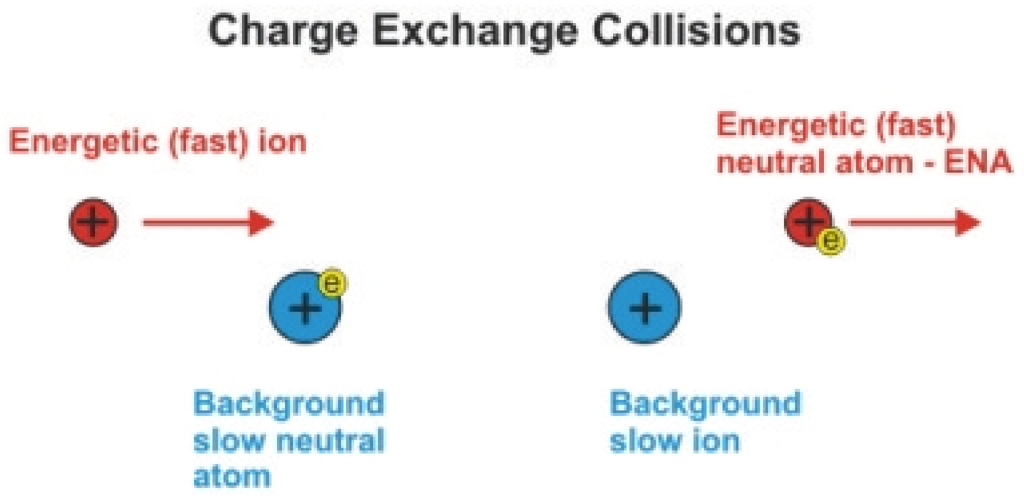
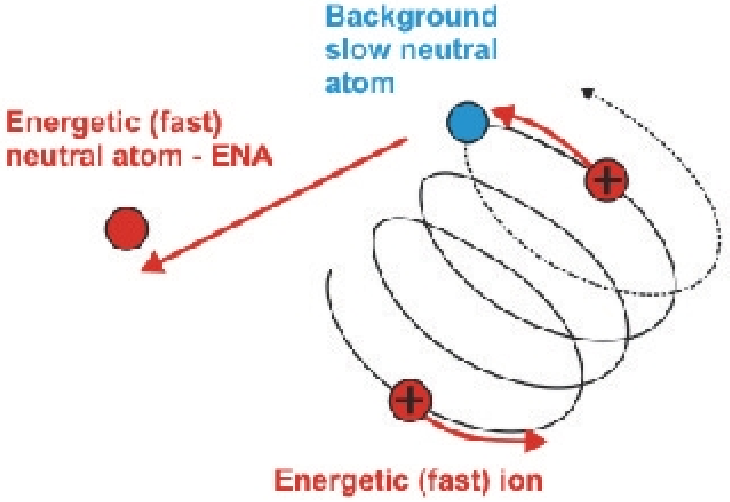
An axisymmetric model of the plume and backflow contamination from an ion-thruster plume is presented. Components included are primary beam ions, neutral propellant efflux, thermal propellant ions created mainly by charge-exchange collisions between primary beam ions and neutral propellant, nonpropellant efflux sputtered from thruster components, and neutralizing electrons. The plasma hybrid particle-in-cell technique is applied





https://www.astronauticsnow.com/ENA/index.html

Electron models: isothermal (Boltzmann)

$$m_e\left(\frac{\partial \mathbf{u}_e}{\partial t} + \mathbf{u}_e \cdot \nabla \mathbf{u}_e\right) = e\nabla\phi - \frac{1}{n_e}\nabla p_e$$

$$\mathbf{u}_e \to 0;\ p_e = n_e k T_e;\ T_e = T_{e0}$$

$$e\nabla\phi = \frac{kT_{e0}}{n_e}\nabla n_e \to n_e = n_{e0}\exp\left(\frac{\phi - \phi_0}{kT_{e0}}\right)$$ n_{e0} is the density at ϕ_0

What are the consequences of this model?

- Electrons at constant temperature → Instant thermalization → infinite number of collisions
- Not cooling down when density reduces (plasma expansion)
- Electrons instantaneously response to changes in electric field

Electron models: polytropic

$$m_e\left(\frac{\partial \mathbf{u}_e}{\partial t}+\mathbf{u}_e\cdot\nabla\mathbf{u}_e\right)=e\nabla\phi-\frac{1}{n_e}\nabla p_e$$

$$\mathbf{u}_e\to 0;\ p_e=n_ekT_e;\ \frac{T_e}{T_{e0}}=\left(\frac{n_e}{n_{e0}}\right)^{\gamma_e-1}$$

$$e\nabla\phi=\frac{k}{n_e}\nabla n_eT_e\to n_e=n_{e0}\left[1+\frac{\gamma_e-1}{\gamma_e}\frac{e\left(\phi-\phi_0\right)}{kT_{e0}}\right]^{\frac{1}{\gamma_e-1}}$$

n_{e0} and T_{e0} are the density and temperature at ϕ_0

What are the consequences of this model?

- New unknown in the system $1<\gamma_e<5/3$
  - Limits between isothermal and adiabatic descriptions
- Electrons instantaneously response to changes in electric field

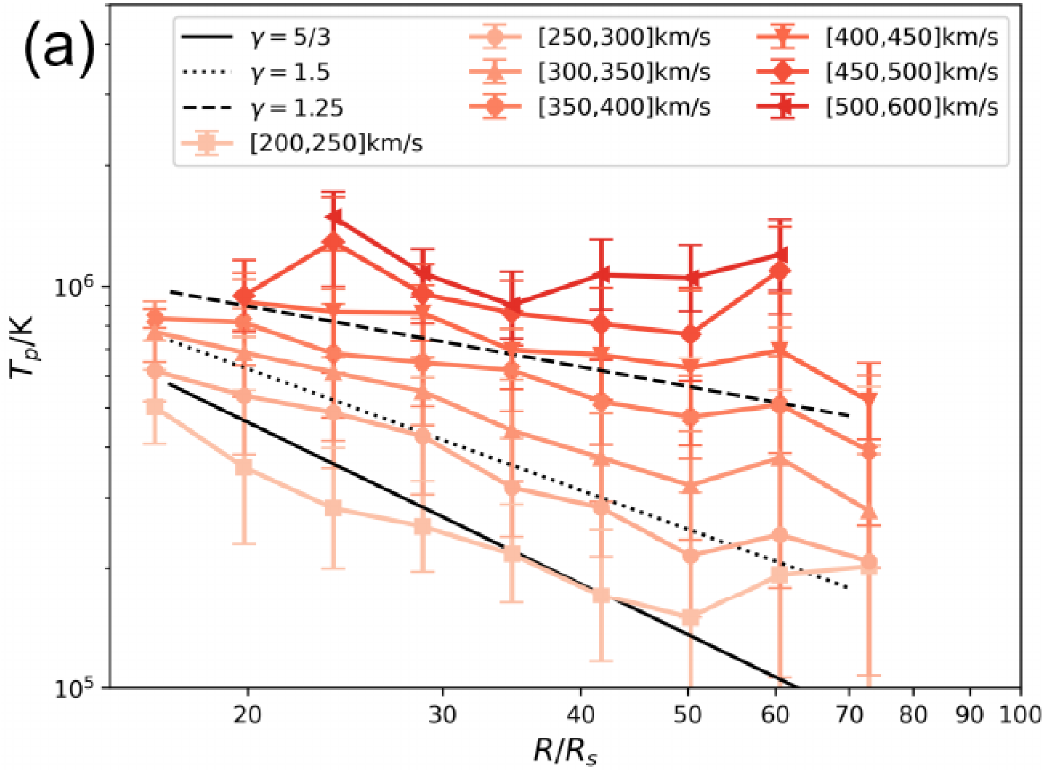
These models can also be applied to ions

Application: Solar wind models

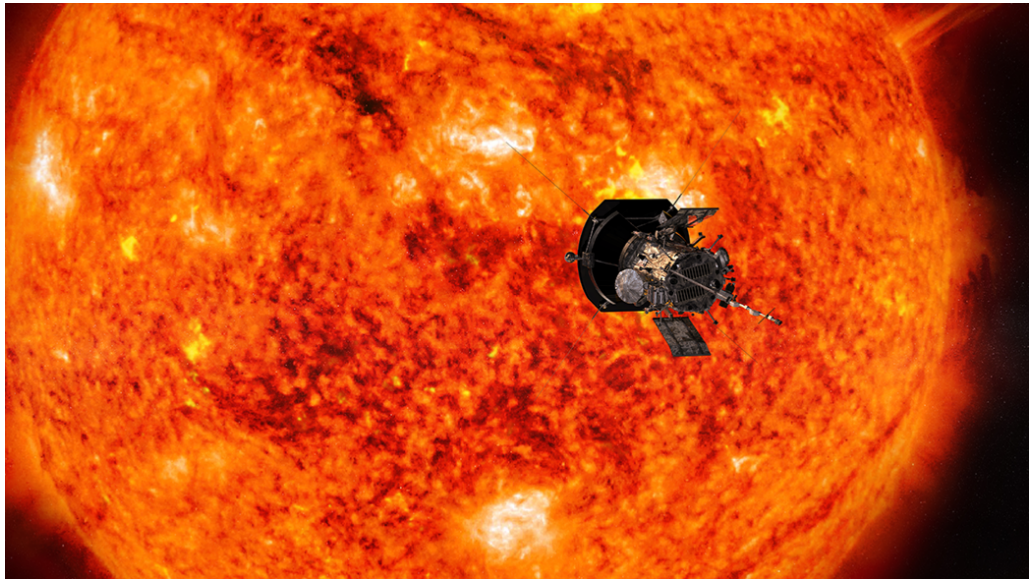
Protons and electrons in the solar wind can be described as polytropic fluid species.

Current observations by the Parker Solar Probe (PSP) (and previous missions like Helios) allow to compare models and measurements of the solar win for distances < 1 au.

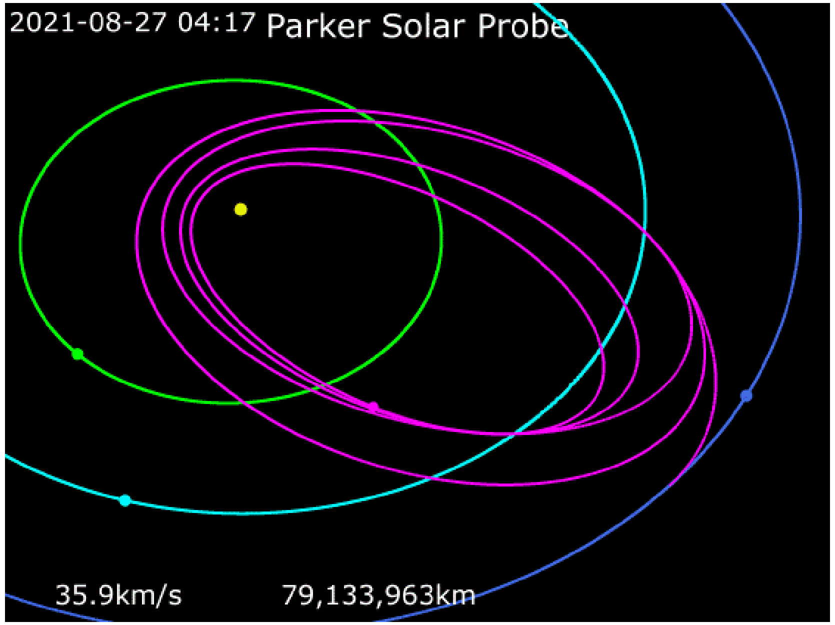
- Minimum distance of 9.86 solar radii (6.9e6 km)
- Maximum speed of 191 km s^{-1} (6.37e-4 c). **Fastests human-made object!**



Phys. Plasmas 29, 122901 (2022); doi: 10.1063/5.0124703

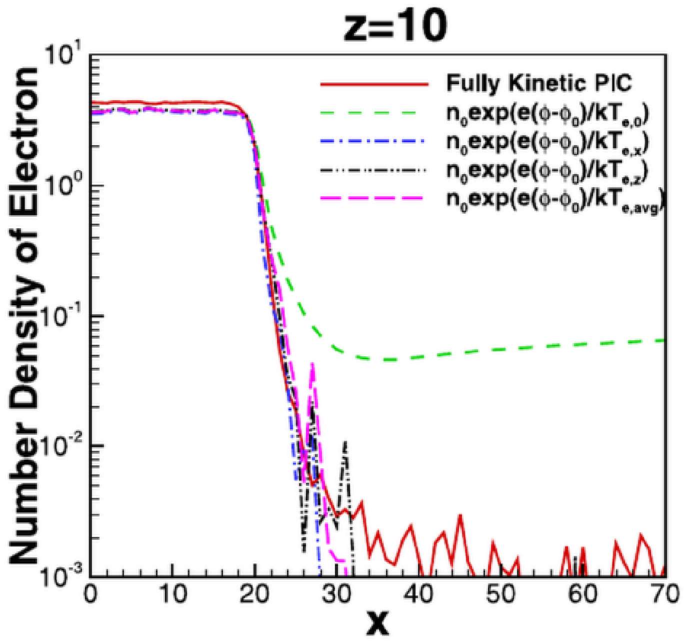


An artist’s concept shows Parker Solar Probe approaching the Sun. NASA/Johns Hopkins APL/Steve Gribben



Application: Plasma expansion and wakes

- Isothermal and polytropic fluid electrons are used in plasma expansion models.
- This allows to save computational time.
- **BUT:** Is the assumption of electrons behaving as a fluid correct? Isohthermal or polytropic? Which polytropic exponent?
  - Well, we fully don't know, but it seems it is not correct.



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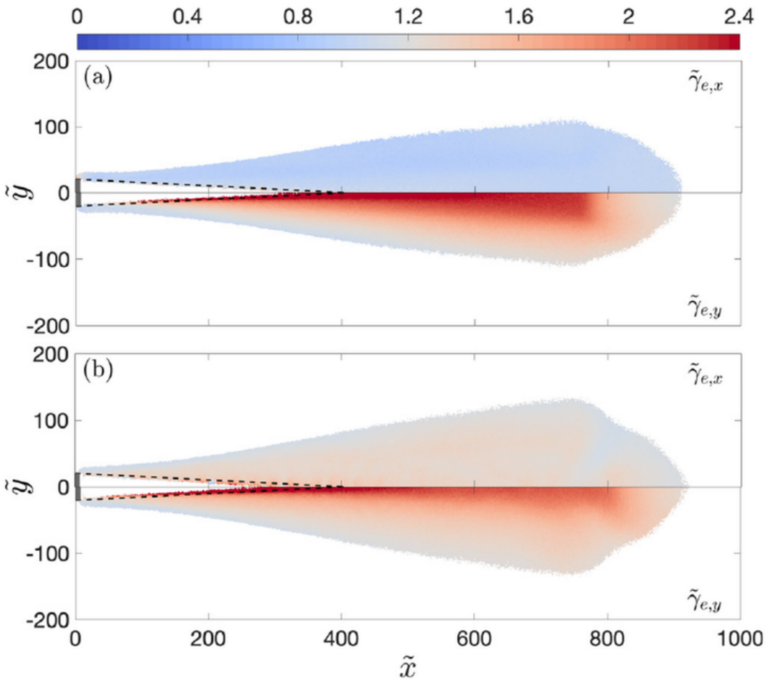

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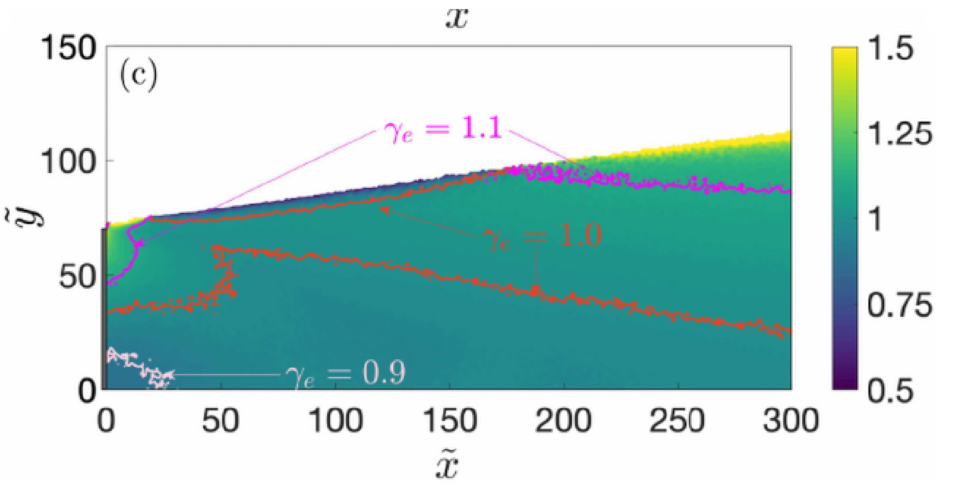

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