



Máster Universitario en Ingeniería Aeronáutica

The Space Environment

Neutral gases versus plasmas. Kinetic Theory

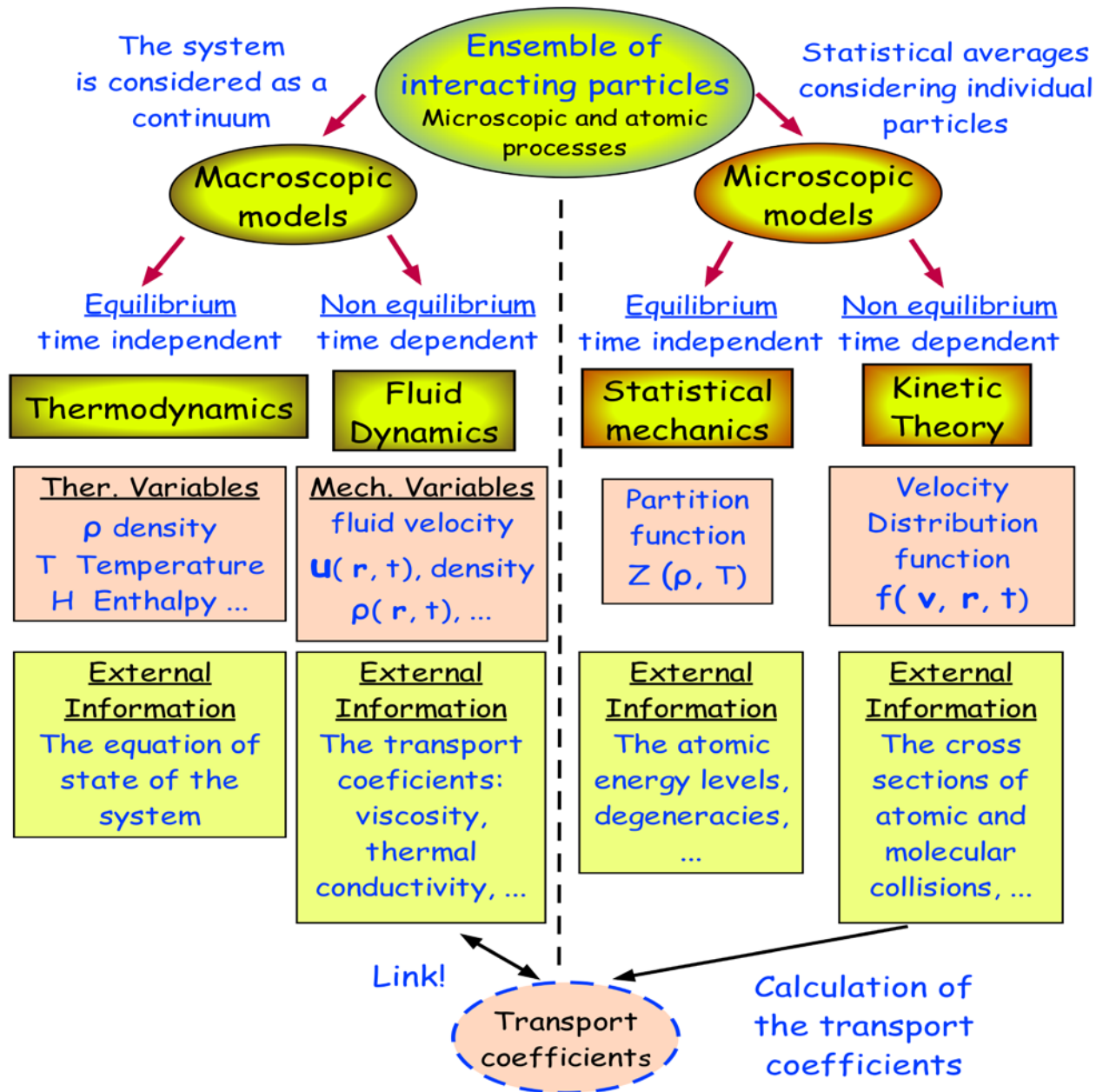
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EL Entorno Espacial. MUIA. *PLASMALAB-ETSIAE-UPM*.

Materia y Página web de la Asignatura basada en la web personal del Prof. Dr. L. Conde:

<https://plasmalab.aero.upm.es/~lcl/EntornoEspacial/>

Plasma models: from particle motions to fluid description:



From single particle motions to fluid view

A first approach:

A) to apply the system *fluid description* (continuity, Navier-Stokes,..., **MHD**) but adding the electromagnetic forces effects on a plasma, for momentum and energy equations to obtain:

In contrast: $n(\mathbf{r}, t), \mathbf{u}(\mathbf{r}, t), \mathbb{P}(\mathbf{r}, t) \dots$

B) Apply *elementary dynamics* to any single particle (conservation laws, force on the particle...) providing individual motions:

$$\frac{1}{2}mv^2 + q\phi(x, t) = E_0; \quad m\mathbf{r} \times \mathbf{v} = \mathbf{L}_0 \dots$$

- A) Losses of information related to particles velocity, *collective effects* can mask microscopic and important physics, misleading results...non-local description.
- B) Too simple, losses of the collective effects (self-consistent forces) leading to very local descriptions of a plasma.
- C) **Several intermediate descriptions** are needed by virtue of the plasma system scenario. It is desirable to account for collective phenomena in suitable time and spatial scales.

Collision parameters, the key

- From microscopic description of particles close the thermodynamic equilibrium :

a distribution function is derived $f(v)$, from it, macroscopic magnitudes can be obtained by averaging calculations, providing a direct relation to macroscopic description (e.g. Temperature is related to average kinetic energy)

$$PV = n_{mol}RT = \frac{N}{N_A}RT = Nk_B T \Rightarrow$$

$$g(P, V, T, N) = PV - Nk_B T = 0 \text{ y } P = nk_B T$$

$$n = \frac{N}{V}, \text{ number density } \left(n_s = \frac{dN_s}{dV}, \text{ plasma specie } s = e, i, a \right)$$

- Microscopic description: particles distributed in velocity and in space (phase space)
- The key for a microscopic view is the mean-free-path , a statistical concept, intuitively, for a test particle a collision with a fixed particle b , this distance would be: (see. Ref. 2, Knudsen number)

$$\left. \begin{aligned} \lambda_c &= \frac{\text{Travelled length}}{\text{collision number}} = \frac{v\Delta t}{\sigma_{ab}n_b v\Delta t} = \frac{1}{\sigma_{ab}n_b} \\ \nu_c &= \frac{1}{\tau_c} = \sigma_{ab}n_b v, \text{ e.g.. if } v = \sqrt{k_B T / m} \end{aligned} \right\}$$

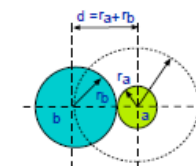


Figure 2.3: The area within the dotted circle is the total cross section σ_{ab} for colliding particles.

Collisions (Irreversibility conditions, microscopic)

A first question: **Is it possible to study plasmas as they were neutral gases**

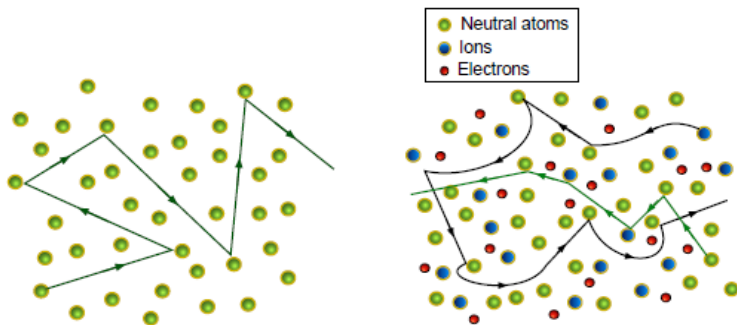
Moreover: Can we deal plasmas as system in (or close to) thermodynamic equilibrium?

A neutral gas reached thermal equilibrium after particles microscopic interactions called, in general, collisions:

- Elastic collisions dominate (binary, instantaneous forces)
- Short-range intense forces.
- Random or zig-zag particle motions.

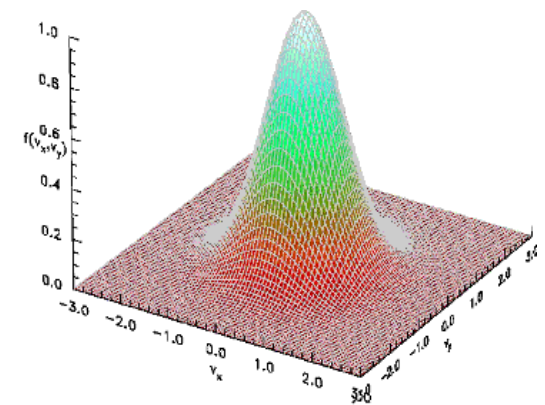
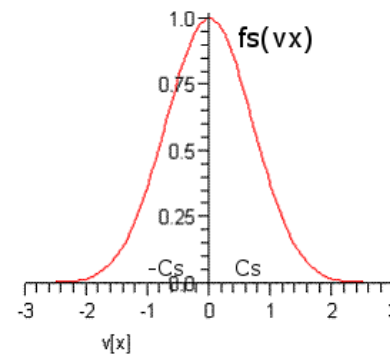
But, in plasmas, charge particles interact by Coulombian forces :

- Smooth trajectories and multiple particle interaction
- Coulomb forces are large-range and weak ones.



(a) The random motion of a neutral atom into its parent gas where trajectories are straight paths.

(b) The random motion of an ion and a neutral atom into a partially ionized gas.



Equilibrium. Neutral gas is microscopically characterized by a statistical description, distribution. Macroscopically, described by a **State Equation** $g(P, V, T, N) = 0$

Statistical description is needed . Remember neutral ideal gas

Gas ideal. Interpretación por teoría cinética.

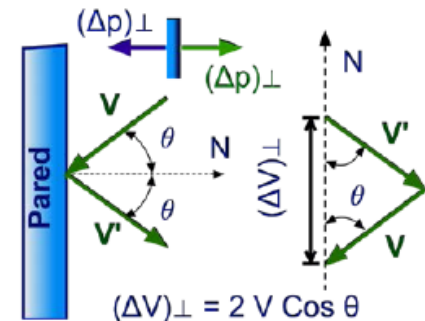
Cálculo de **energía cinética** media en gas ideal con N partículas puntuales no interactuantes en volumen V y *movimiento aleatorio*. La cantidad de partículas dirigidas perpendicularmente hacia la pared ($v_z > 0$, la mitad) de área S del recipiente en tiempo δt cuando ocupan volumen δV *adyacente* es: (la mitad van hacia pared)

$$\frac{\delta N}{\delta V} = \frac{N}{V} \Rightarrow N_{\perp} = \frac{1}{2} \delta N = \frac{1}{2} N \frac{\delta V}{V} = \frac{1}{2} \frac{N}{V} S \delta z = \frac{1}{2} \frac{N}{V} S v_{\perp} \delta t$$

El intercambio neto de momento lineal por colisiones con pared:

$$\delta P_{\perp} = N_{\perp} (m_o v_{\perp} - (-m_o v_{\perp})) = \frac{N}{V} m_o S v_{\perp}^2 \delta t \equiv \frac{N}{V} m_o S v_z^2 \delta t$$

Y la presión ejercida:
$$p = \frac{F_z}{S} = \frac{\delta P_{\perp} / \delta t}{S} = \frac{N}{V} m_o v_z^2$$



En promedio (**equipartición** de energía):

$$\langle v^2 \rangle = \langle v_x^2 + v_y^2 + v_z^2 \rangle = 3 \langle v_z^2 \rangle \rightarrow \langle v_z^2 \rangle = \frac{1}{3} \langle v^2 \rangle$$

Da

$$pV = N m_o \langle v_z^2 \rangle = \frac{2}{3} N \left(\frac{1}{2} m_o \langle v^2 \rangle \right) = \frac{2}{3} N \langle E_c \rangle \equiv nRT = \frac{N}{N_A} RT = N k_B T$$

donde $k_B = R/N_A$ es la *constante de Boltzmann* , y las energías cinética total y media por partícula:

$$E_c = \frac{1}{2} N m_o \langle v^2 \rangle = N \left(\frac{3}{2} k_B T \right) \rightarrow \langle E_c \rangle = \frac{3}{2} k_B T ; k_B = \frac{R}{N_A} \simeq 1,38 \cdot 10^{-23} \text{ JK}^{-1}$$

Ionosphere data

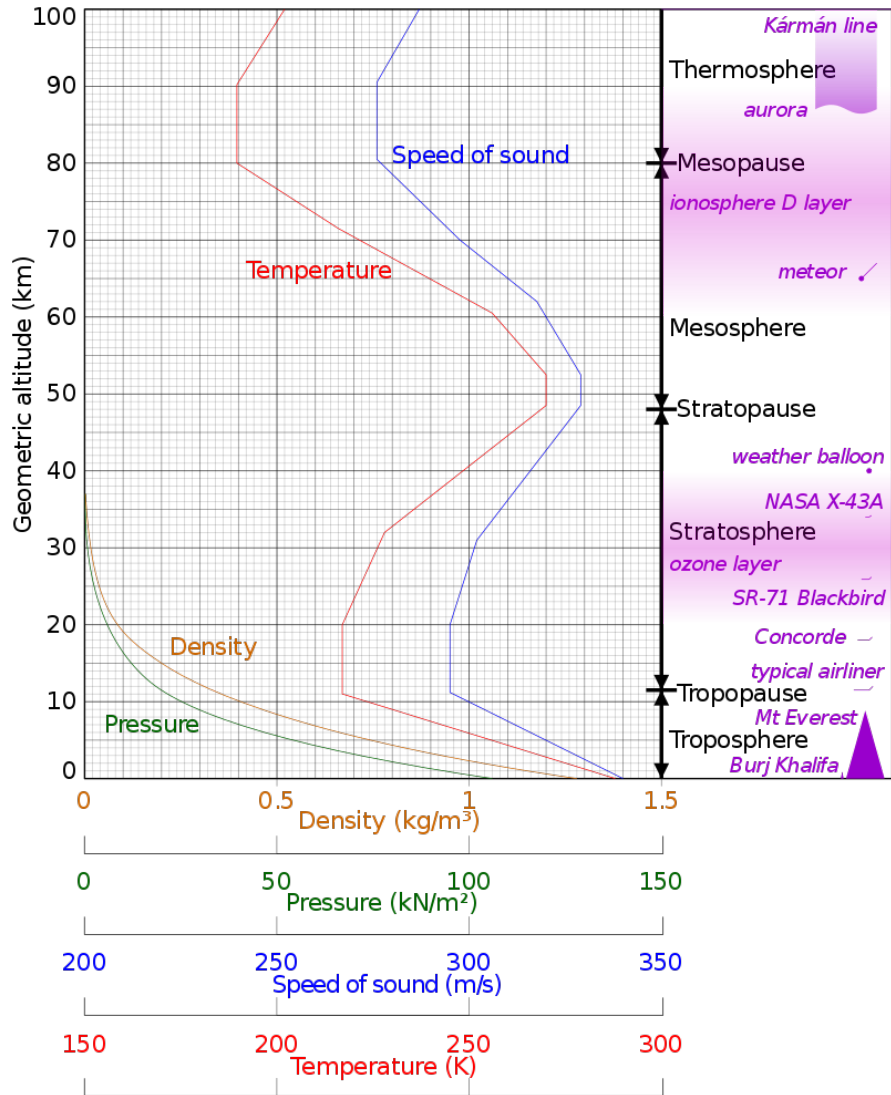
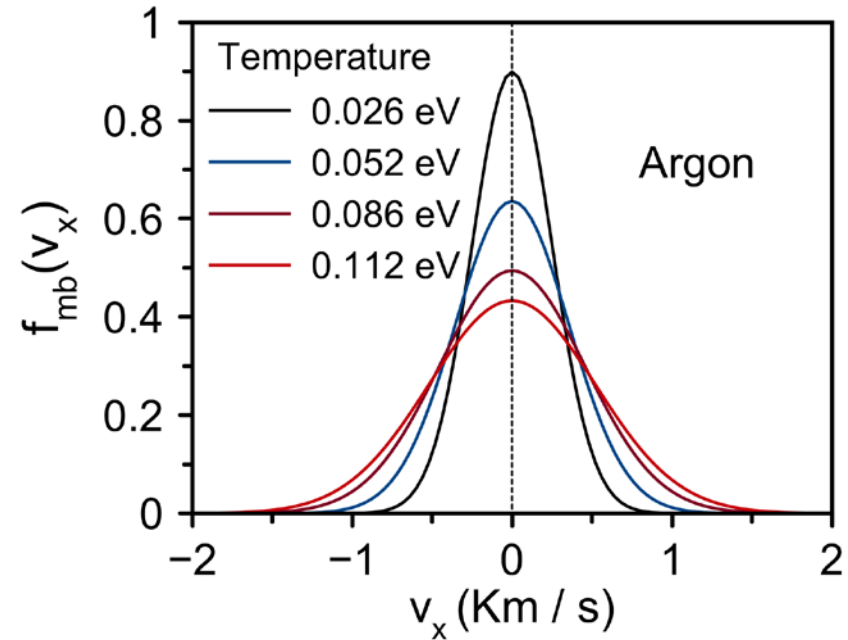


Table 1.1. The characteristics of ionospheric plasmas for different altitudes from reference [4]. The temperatures of plasma species (neutral gas T_a , ions T_i and electrons T_e) are in Kelvin and \bar{m}_i is the average molecular mass of ions in atomic mass units. The plasma density is n_o and λ_c



Nature of collisions defines gas time and spatial scales

Neutral vs ionized gases will differ in time and space evolving scales, T and L , larger compared to collisional microscopic scales.

$$L \gg \lambda_c$$

$$T \gg \tau_c$$

The Maxwell-Boltzmann homogeneous distribution $f(v)$ is reached by a neutral gas, it can be considered as the probability of finding a particle with velocity in the range v and $v+dv$, or, alternatively, the number of particles dn in this range:

(see Material Auxiliar Ref. 1.3 and Ref. 2.1)

$$\left\{ \begin{array}{l} f(v_x) = n_x \sqrt{\frac{m}{2\pi k_B T}} e^{-\frac{mv_x^2}{2k_B T}}; \quad \int_{-\infty}^{\infty} f(v_x) dv_x = n_x \\ f(\mathbf{v}) = n_0 \left(\sqrt{\frac{m}{2\pi k_B T}} \right)^3 e^{-\frac{m|\mathbf{v}|^2}{2k_B T}}; \quad \int f(\mathbf{v}) d^3v = n_0 \end{array} \right\} E = \frac{1}{2} m |\mathbf{v}|^2$$

And, if the gas is under a field action with potential energy Φ (changing the kinetic energy E by the mechanical energy) we get the Maxwell-Boltzmann distribution:

$$f(\mathbf{r}, \mathbf{v}) = N_0 \left(\sqrt{\frac{m}{2\pi k_B T}} \right)^3 e^{-\frac{m|\mathbf{v}|^2/2 + \Phi(\mathbf{r})}{k_B T}}; \quad \int f d^3v = n_0(\mathbf{r}) = N_0 e^{-\frac{\Phi(\mathbf{r})}{k_B T}}; \quad \int f d^3v d^3r = N_0$$

Maxwell-Boltzmann Distribution

The Maxwell-Boltzmann probability distribution

$$f_{mb}(v) = \left(\frac{m}{2\pi k_B T} \right)^{3/2} e^{-mv^2/2k_B T}$$

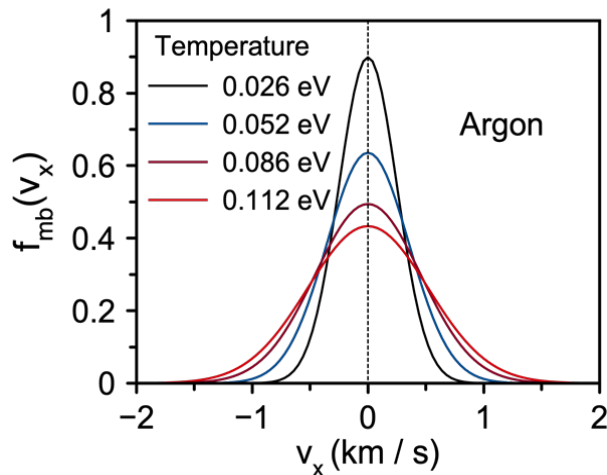
$$g_{mb}(E) = \frac{2\sqrt{E} e^{-E/k_B T}}{\sqrt{\pi} (k_B T)^{3/2}}$$

$$\int_0^{\infty} g_{mb}(E) dE = 1$$

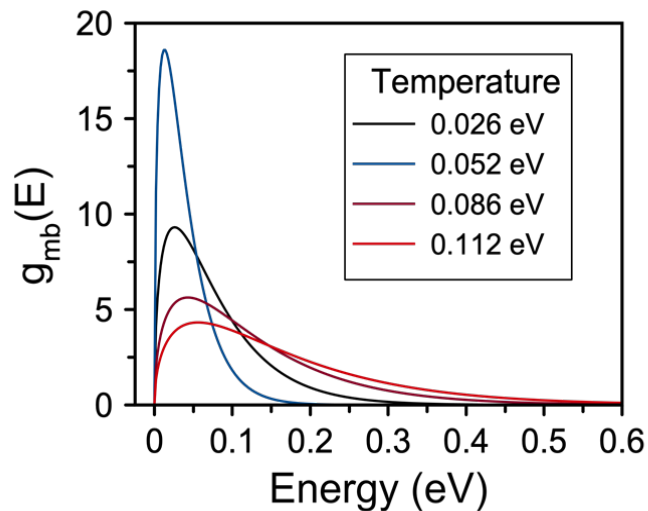
- Strictly speaking only applies to systems in thermal equilibrium (uniform n_o and T).
- The function $f_{mb}(v)$ is the **probability density** and $dP = f_{mb}(v) d^3v$ the probability of finding one particle within the volume (d^3v) centered around the velocity v .
- The function $g_{mb}(E)$ is the **probability density** and $dP = g_{mb}(E) dE$ the probability of finding one particle with kinetic energy in the $(E, E + dE)$ range.
- The MB **velocity distribution function** gives the $dN = n_o f_{mb}(v) d^3r d^3v$ number of particles within the $(d^3r d^3v)$ elementary volume located at the (r, v) point in the phase space.

Represented in one dimension:

$$f_{mb}(v_x) = \left(\frac{m}{2\pi k_B T} \right)^{1/2} e^{-mv_x^2/2k_B T}$$



Represented as a function of the kinetic energy.



Maximum of $g_{mb}(E)$ defines:

- Most probable energy: $E = \frac{k_B T}{2}$
- Most probable velocity: $v = \sqrt{\frac{2k_B T}{m}}$

Application: Field **shielding**, Debye Length as fundamental plasma spatial scale

- Let us assume an equilibrium Maxwellian plasma. For a point charge in a local perturbation of plasma neutrality, the local electric potential satisfies: See Ref. 3). We can see that the E field is shielded from the any point charge (the individual field vanishes is a short distance).

In the initial plasma equilibrium, we introduce^(*) a small perturbation $\delta\rho_c = q \delta(\mathbf{r})$ in the electric charge

$$\left. \begin{array}{l} \delta\rho_{ext} = q \delta(\mathbf{r}) \\ \delta\rho_{sp}(\mathbf{r}) = e [n_i(\mathbf{r}) - n_e(\mathbf{r})] \end{array} \right\} \left. \begin{array}{l} \mathbf{E}(\mathbf{r}) \simeq \mathbf{E}_0 + \mathbf{E}_1(\mathbf{r}) \\ \mathbf{E}_0 \simeq 0 \text{ (plasma equilibrium)} \end{array} \right\} \begin{array}{l} \mathbf{E}_1(\mathbf{r}) = -\nabla\varphi_1(\mathbf{r}) \\ \mathbf{E}_1(\mathbf{r}) \text{ is the } \textit{perturbed electric field} \\ \text{governed by the Poisson equation} \end{array}$$

$$\nabla \cdot \mathbf{E}_1 = -\nabla^2 \varphi_1 = \frac{\delta\rho_c + \delta\rho_{sp}}{\epsilon_0} \qquad \nabla \cdot \mathbf{E}_1 = \frac{q}{\epsilon_0} \delta(\mathbf{r}) + \frac{e}{\epsilon_0} [n_{i1}(\mathbf{r}) - n_{e1}(\mathbf{r})]$$

In a Maxwellian plasma in equilibrium the $n_e(\mathbf{r})$ and $n_i(\mathbf{r})$ densities are given by, $n_\alpha(\mathbf{r}) = n_o \exp\left(\pm \frac{e \varphi_1(\mathbf{r})}{k_B T_\alpha}\right)$

Small amplitude perturbations of the charge/electric field means that *thermal energy* $k_B T$ dominates over the *electrostatic energy* $|e\varphi_1(\mathbf{r})|$ in this case we said this is an *ideal plasma* then we can approximate for ions and electrons

$$\left. \begin{array}{l} \left| \frac{e \varphi_1(\mathbf{r})}{k_B T_\alpha} \right| \ll 1 \quad n_\alpha(\mathbf{r}) \simeq n_o \left(1 \pm \frac{e \varphi_1(\mathbf{r})}{k_B T_\alpha} \right) \\ -\nabla^2 \varphi_1(\mathbf{r}) = \frac{1}{\epsilon_0} \left[\delta\rho_{ext} + \frac{e^2 n_o}{k_B T_e} \varphi_1(\mathbf{r}) + \frac{e^2 n_o}{k_B T_e} \varphi_1(\mathbf{r}) \right] \end{array} \right\}$$

^(*) Details of this derivation in the note entitled *The ideal Maxwellian plasma* available in the supplementary material lectures.

- The electric field perturbation decays exponentially from any test point charge q in a quasi-equilibrium neutral plasma**

Application: Debye Length as fundamental plasma spatial scale II

- For a point charge in a local perturbation of plasma neutrality, the local electric potential satisfies: See Ref. 3)

$$\nabla^2 \phi = -\frac{\delta \rho}{\epsilon_0} = -\frac{1}{\epsilon_0} \left[\delta \rho_{ext} + \frac{e^2 n_o}{k_B T_i} \phi(\mathbf{r}) + \frac{e^2 n_o}{k_B T_e} \phi(\mathbf{r}) \right]$$

For a small deviation from equilibrium, gives the characteristic Debye Length:

$$\left(\nabla^2 - \frac{1}{\Lambda_D^2} \right) \phi(\mathbf{r}) = -\frac{q}{\epsilon_0} \delta(\mathbf{r}) \quad \frac{1}{\Lambda_D^2} = \frac{1}{\lambda_{Di}^2} + \frac{1}{\lambda_{De}^2}$$

- Or electron and ion Debye lengths and potential (shielded field) solution:

$$\lambda_{Di} = \sqrt{\frac{\epsilon_0 k_B T_i}{e^2 n_o}} \quad ; \quad \lambda_{De} = \sqrt{\frac{\epsilon_0 k_B T_e}{e^2 n_o}}$$

$$\frac{d^2 \phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} - \frac{\phi}{\Lambda_D^2} = -\frac{q}{\epsilon_0} \delta(r) \quad \phi_D(r) = \frac{q}{4\pi\epsilon_0} \frac{e^{-r/\Lambda}}{r}$$

- The electric field perturbation decays exponentially from any test point charge q in a quasi-equilibrium plasma state: **No large E field can be sustained inside a conductive medium**

A first (exact and useless) distribution :

An **exhaustive and complete approach** would give **the useless** (unreachable) **peaked abrupt distribution** for N particles in deterministic motion: **(Klimontovich distribution)** knowing every point particle deterministic, Newtonian, position and velocity at time t , we would have:

$$f_K(\mathbf{r}, \mathbf{v}, t) = \sum_{i=1}^N \delta[\mathbf{r} - \mathbf{r}_i(t)] \delta[\mathbf{v} - \mathbf{v}_i(t)]$$

Giving the macroscopic charge and electric current densities of punctual elementary charged particles sets : **(check physical units !!)**

$$\left. \begin{aligned} \rho(\mathbf{r}, t) &= \sum_i q_i \delta(\mathbf{r} - \mathbf{r}_i) \\ \mathbf{J}(\mathbf{r}, t) &= \sum_i q_i \mathbf{v}_i \delta(\mathbf{r} - \mathbf{r}_i) \end{aligned} \right\} \text{con} \left\{ \begin{aligned} \int f_K(\mathbf{r}, \mathbf{v}, t) d^3\mathbf{v} &= \sum_{i=1}^N \delta(\mathbf{r} - \mathbf{r}_i) = n(\mathbf{r}, t) \\ \int f_K(\mathbf{r}, \mathbf{v}, t) d^3\mathbf{v} d^3\mathbf{r} &= \sum_{i=1}^N 1 = N \end{aligned} \right.$$

$$\begin{aligned} \nabla \cdot \mathbf{E} &= \frac{\rho_c}{\epsilon_0} & \nabla \wedge \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} &= 0 & \nabla \wedge \mathbf{B} &= \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} + \frac{\mathbf{J}_c}{\epsilon_0 c^2} \end{aligned}$$

$$E_{jm}: \quad \rho_e = \frac{\sum_{i=1}^{\Delta N} q_e}{\Delta V} = q_e \frac{\Delta N}{\Delta V} = q_e n_e ; \quad \mathbf{J}_e(\mathbf{r}, t) = \frac{\sum_{i=1}^{\Delta N} q_e \mathbf{v}_i}{\Delta V} = \frac{\sum_{i=1}^{\Delta N} q_e \mathbf{v}_i}{\Delta N} \frac{\Delta N}{\Delta V} = q_e \langle \mathbf{v} \rangle_e n_e = q_e \mathbf{u}_e n_e = q_e \int \mathbf{v} f_e(\mathbf{v}, \mathbf{r}, t) d\mathbf{v}$$

One distribution for each species: electrons, ions and neutrals, coupled to Maxwell Equations in Electr.

$$\rho_\alpha(\mathbf{r}, t) = m_\alpha n_\alpha(\mathbf{r}, t) \quad \rho_c(\mathbf{r}, t) = \sum_\alpha \rho_{e\alpha} = \sum_\alpha q_\alpha n_\alpha(\mathbf{r}, t) \quad \alpha = e, i, a$$

$$\mathbf{J}_c(\mathbf{r}, t) = \sum_\alpha \mathbf{J}_{e\alpha} = \sum_\alpha q_\alpha n_\alpha \mathbf{u}_\alpha = q_\alpha \int_{-\infty}^{+\infty} f_\alpha(\mathbf{v}, \mathbf{r}, t) \mathbf{v} d\mathbf{v}$$

Extension: Information given by f and macroscopic magnitudes

- The **most important macroscopic measurables** (magnitude measurements), for each plasma species α , derived from the distribution function are averages of velocity functions $H(\mathbf{v})$ (usually powers of velocity components) in the form: (Definitions) $\alpha = \text{electrons } e, \text{ ions } i, \text{ and neutrals}$

$$\langle H \rangle_{\alpha}(\mathbf{r}, t) = \frac{1}{n_{\alpha}} \int H(\mathbf{v}, \mathbf{r}, t) f_{\alpha}(\mathbf{v}, \mathbf{r}, t) d\mathbf{v} \Leftrightarrow n_{\alpha} \langle H \rangle_{\alpha} = \int H(\mathbf{v}, \mathbf{r}, t) f_{\alpha}(\mathbf{v}, \mathbf{r}, t) d\mathbf{v}$$

$$\text{number density: } n_{\alpha} = \int f_{\alpha}(\mathbf{v}, \mathbf{r}, t) d\mathbf{v}$$

$$\text{Fluid velocity: } \mathbf{u}_{\alpha}(\mathbf{r}, t) = \langle \mathbf{v} \rangle = \frac{1}{n_{\alpha}} \int \mathbf{v} f_{\alpha}(\mathbf{v}, \mathbf{r}, t) d\mathbf{v}$$

$$\text{Temperature: } \frac{3}{2} k_B T_{\alpha}(\mathbf{r}, t) = \left\langle \frac{1}{2} m_{\alpha} |\mathbf{v} - \mathbf{u}|^2 \right\rangle = \frac{1}{n_{\alpha}} \int \frac{1}{2} m_{\alpha} |\mathbf{v} - \mathbf{u}|^2 f_{\alpha}(\mathbf{v}, \mathbf{r}, t) d\mathbf{v}$$

- And the derived ones (observe that T and energy transport, heat, are measured accounting for thermal fluctuations around fluid velocity \mathbf{u}):

$$\text{mass density: } \rho = \sum_{\alpha} m_{\alpha} n_{\alpha}, \text{ charged density: } \rho_q = \sum_{\alpha} q_{\alpha} n_{\alpha}$$

$$\text{Particle flux: } \mathbf{\Gamma} = \sum_{\alpha} \mathbf{\Gamma}_{\alpha} = \sum_{\alpha} n_{\alpha} \langle \mathbf{v} \rangle_{\alpha}$$

$$\text{Electric current density: } \mathbf{J}(\mathbf{r}, t) = \sum_{\alpha} q_{\alpha} n_{\alpha} \langle \mathbf{v} \rangle_{\alpha} = \sum_{\alpha} q_{\alpha} n_{\alpha} \mathbf{u}_{\alpha}$$

$$\text{Heat flux: } \mathbf{q}(\mathbf{r}, t) = n_{\alpha} \left\langle \frac{1}{2} m_{\alpha} |\mathbf{v} - \mathbf{u}|^2 (\mathbf{v} - \mathbf{u}) \right\rangle, \text{ Entropy: } S = -k_B \iint f \ln f d\mathbf{v} d\mathbf{r}$$

A differential evolution equation for f is needed

Approach (theoretical interest only): The *Klimontovich* distribution is in practice replaced by a formal *smooth density* $f(\mathbf{r}, \mathbf{v}, t)$ in phase space point “particles” at position $\mathbf{q}=(\mathbf{r}, \mathbf{v})$ of a **six-dimensional** space, understood as a number of points in a \mathbf{q} -space 6-Dim, still small, volume:

$$f(\mathbf{r}, \mathbf{v}, t) = \frac{1}{\Delta \mathbf{r} \Delta \mathbf{v}} \int_{\Delta \mathbf{r}} d\mathbf{r} \int_{\Delta \mathbf{v}} d\mathbf{v} f_K = \frac{N_p(\mathbf{r}, \mathbf{v}, t)}{\Delta \mathbf{r} \Delta \mathbf{v}}$$

that generates a smooth function f as the (number) **density** of particles inside a small six-dimensional volume element centered at position $\mathbf{q}=(\mathbf{r}, \mathbf{v})$.

NOTE: This approach is similar to that one expressing mass density ρ as a continuous smooth function of position of a small volume containing ΔN particles, instead of a distribution of point masses by a set Dirac Delta densities $\rho(\mathbf{r}) = \sum_i^{\Delta N} m_i / \Delta V$

As a density $f(\mathbf{r}, \mathbf{v}, t)$, has to satisfy a **continuity equation** if no particles are created or destroyed:

$$\frac{df}{dt} \equiv \frac{\partial f}{\partial t} + \frac{\partial}{\partial \mathbf{q}} \cdot (\mathbf{U}_q f) = \frac{\partial f}{\partial t} + \text{div}(\mathbf{U}_q f) = 0; \quad \mathbf{U}_q = (\dot{\mathbf{r}}, \dot{\mathbf{v}}) = (\mathbf{v}, \mathbf{F}/m)$$

In particular:

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \cdot (f \mathbf{v}) + \frac{\partial}{\partial \mathbf{v}} \cdot (f \mathbf{a}) = 0$$

Where \mathbf{r} and \mathbf{v} , as components of $\mathbf{q}=(\mathbf{r}, \mathbf{v})$, are INDEPENDENT variables.

THERE is a distribution function for each plasma species ;;;;

Kinetic: A very general formulation is possible knowing the time evolution of f , valid for **Inhomogeneous, anisotropic and non-equilibrium plasmas.**

Kinetic Theory: general and basic for other approaches

For the species density f in a point of the configuration space \mathbf{r} , as for the density of $dN=n d\mathbf{r}$, in velocity space, it can be defined a similar number density being proportional to volume element $d\mathbf{r}$ and function of \mathbf{r} , t and \mathbf{v} as $f(\mathbf{r}, \mathbf{v}, t) d\mathbf{r}$ of phase space

The number of points dN_p in the volume $d\mathbf{r} d\mathbf{v}$ of phase space with velocities lying between \mathbf{v} and $\mathbf{v} + d\mathbf{v}$ and position between \mathbf{r} and $\mathbf{r} + d\mathbf{r}$ is $dN_p = f(\mathbf{r}, \mathbf{v}, t) d\mathbf{r} d\mathbf{v} = f(\mathbf{r}, \mathbf{v}, t) d^3r d^3v = f(\mathbf{q}, t) d^6q$
Being the number density of real particles in physical volume element and $dn = f(\mathbf{r}, \mathbf{v}, t) d\mathbf{v} = f d^3v$ giving the density n and the number of particles as

$$n(\mathbf{r}, t) = \int f(\mathbf{r}, \mathbf{v}, t) d\mathbf{v} \quad \text{and} \quad N = \int n(\mathbf{r}, t) d\mathbf{r}$$

$f(\mathbf{r}, \mathbf{v}, t)$ is the **spatial-velocity distribution function**, it can be also understood as a probability density of points in the 6-D \mathbf{r} - \mathbf{v} phase space (then $\int f d\mathbf{r} d\mathbf{v} = 1$)

Kinetic Theory: A very general formulation is possible knowing the time evolution of f , valid for Inhomogeneous, anisotropic and non-equilibrium plasmas.

It is quite general (usually unaffordable) and practical approximate theories are needed

*To do this, it is important to distinguish interactions **in/out** a **Debye Sphere**, to account for:*

microscopic fields simulated by collisional effects dominate in a Debye Sphere, ***whereas macroscopic fields (contribution out of Debye Sphere)*** also enter as a response of the ***collective effects***, also changing them, intrinsic in any plasma

ALL PLASMA DESCRIPTIONS APPEAR IN PLASMA ENVIRONMENT SCENARIOS

Kinetic Theory: general and basic

For the species density in a point of the configuration space \mathbf{r} , as for the density of $n d\mathbf{r}$, in velocity space it is defined a similar number density being proportional to volume element $d\mathbf{r}$ and function of \mathbf{r} , t and \mathbf{v} as $f(\mathbf{r}, \mathbf{v}, t) d\mathbf{r}$.

The number of particles dN in the volume $d\mathbf{r}$ with velocities lying between \mathbf{v} and $\mathbf{v} + d\mathbf{v}$ is

$$dN = f(\mathbf{r}, \mathbf{v}, t) d\mathbf{r} d\mathbf{v} = f(\mathbf{r}, \mathbf{v}, t) dr^3 dv^3 = f(\mathbf{r}, \mathbf{v}, t) d^3r d^3v$$

Finally:

$f(\mathbf{r}, \mathbf{v}, t)$ is the **spatial-velocity distribution function**, it can be also understood as a probability density of points in the 6-D \mathbf{r} - \mathbf{v} phase space.

Kinetic Theory: A very general formulation is possible knowing the time evolution of f , valid for Inhomogeneous, anisotropic and non-equilibrium plasmas. It is quite general (usually unaffordable) and practical approximate theories are needed

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ALL PLASMA DESCRIPTIONS APPEAR IN PLASMA INVIROMENT SCENARIOS

Kinetic Theory : importance of collisions (thermalization/irreversibility)

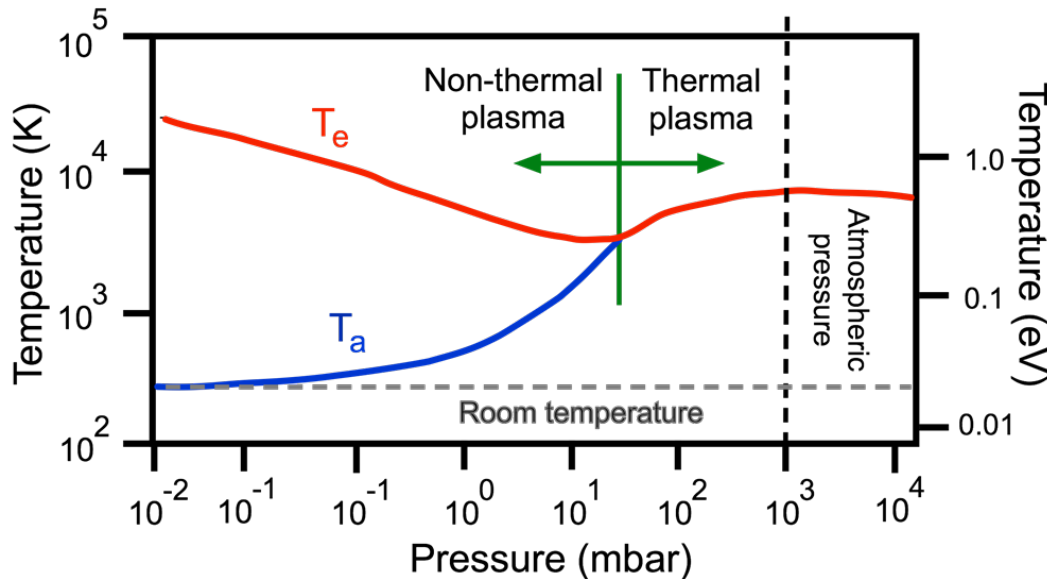
Collisional energy thermalization in a plasma

Energy transference in one elastic collision between particles

$$\left\{ \begin{array}{l} \delta_E = \frac{E_i}{E_a} = \frac{4 m_i m_a}{(m_a + m_i)^2} \cong 1 \\ \delta_E = \frac{E_a}{E_e} = \frac{4 m_e m_a}{(m_a + m_e)^2} \cong \frac{2.2 \cdot 10^{-3}}{Z} \end{array} \right\} \text{ For a complete energy transference } E_e \rightarrow E_a$$

$$N_c \times \delta_E = \frac{E_a}{E_e} \sim 1 \quad \left\{ \begin{array}{l} \text{Helium (Z = 4)} \quad N_c = 1.8 \cdot 10^3 \\ \text{Argon (Z = 40)} \quad N_c = 1.8 \cdot 10^4 \\ \text{Xenon (Z = 132)} \quad N_c = 5.9 \cdot 10^4 \end{array} \right.$$

- The distribution of energy between electrons and heavy species requires of many collisions.



From J. R. Roth. *Industrial plasma engineering*.
Institute of Physics Publishing. Bristol U.K. (1995).

- The *total collision frequency*,

$$\nu_{ea} = \sigma_{ea} n_a n_e u_{ab}$$

characterizes the number of collisions between electrons and neutrals per time and volume unit.

- The transference of energy is proportional to the pressure of the neutral gas.
- We can introduce,
 - Non-thermal plasma* $k_B T_e \gg k_B T_i \cong k_B T_a$
 - Thermal plasma* $k_B T_e \cong k_B T_i \cong k_B T_a$

Práctica: Klimontovich f

Unidades:

- N partículas deterministas con velocidades y posiciones conocidas, del orden del Número de Avogadro
- Densidad numérica n , en partículas / m^3 , o sea, m^{-3}
- La delta de Dirac de \mathbf{r} tiene unidades de m^{-3}
- La delta de Dirac de \mathbf{v} tiene unidades de $(m/s)^{-3}$

Dar las unidades en las que se mide la distribución f , y comprobar que se satisfacen tanto para la f_k como para la Maxwelliana. Observad que:

$$\int_{Dom} f(\mathbf{q}) \delta(\mathbf{q} - \mathbf{q}_0) d^n q = \begin{cases} f(\mathbf{q}_0), & \text{si } \mathbf{q}_0 \in Dom \subseteq R^n \\ 0, & \text{si } \mathbf{q}_0 \notin Dom \end{cases}$$

particularizar para $\mathbf{q} = (\mathbf{r}, \mathbf{v}) \in Dom \subseteq R^{3+3}$ (espacio de fases)

$$f(\mathbf{v}) = n_0 \left(\sqrt{\frac{m}{2\pi k_B T}} \right)^3 e^{-\frac{m|\mathbf{v}-\mathbf{u}|^2}{2k_B T}} \Rightarrow$$

$$\int \mathbf{v} f(\mathbf{v}) d^3 v = n_0 \mathbf{u}$$

$$\frac{1}{2} \int m |\mathbf{v} - \mathbf{u}|^2 f(\mathbf{v}) d^3 v = n_0 \frac{3}{2} k_B T$$

$$\left. \begin{aligned} \rho(\mathbf{r}, t) &= \sum_i q_i \delta(\mathbf{r} - \mathbf{r}_i) \\ \mathbf{J}(\mathbf{r}, t) &= \sum_i q_i \mathbf{v}_i \delta(\mathbf{r} - \mathbf{r}_i) \end{aligned} \right\} \text{con} \begin{cases} \int f_K(\mathbf{r}, \mathbf{v}, t) d^3 \mathbf{v} = \sum_{i=1}^N \delta(\mathbf{r} - \mathbf{r}_i) = n(\mathbf{r}, t) \\ \int f_K(\mathbf{r}, \mathbf{v}, t) d^3 \mathbf{v} d^3 \mathbf{r} = \sum_{i=1}^N 1 = N \end{cases}$$

Práctica. Interpretar las f y calcular n , \mathbf{u} y T

Problem 1) Discuss the meaning of the elementary distributions:

$$f_{\text{beam}}(\mathbf{r}, \mathbf{v}, t) = f_{\text{beam}}(\mathbf{v}) = N_0 \delta(v_x - v_0) \delta(v_y) \delta(v_z)$$

$$f_{\text{shell}}(\mathbf{r}, \mathbf{v}, t) = f_{\text{shell}}(\mathbf{v}) = A \delta(v - v_0)$$

(indication: evaluate the averages of velocity components and modulus)

Compute n , \mathbf{u} and T .

The distribution function $f(\mathbf{r}, \mathbf{v}, t)$ plays the role of density $n(\mathbf{r}, t)$ but **in 6-Dim phase space**:

f is the density of particles related to the number of particles in an elemental 6-Dim volume of phase space.

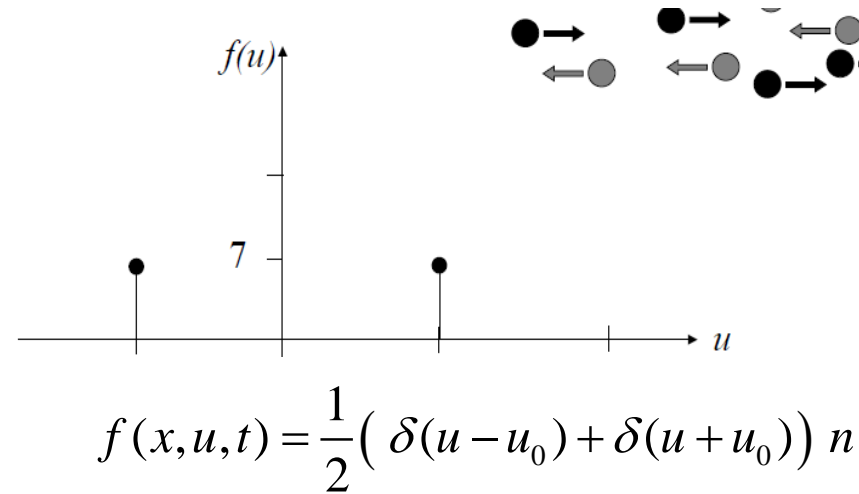
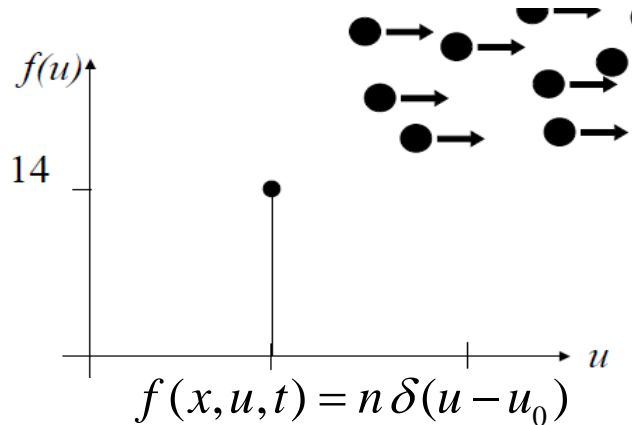
The time evolution of $f(\mathbf{r}, \mathbf{v}, t)$ is described by the **KINETIC THEORY**

The number of particles in **an elemental volume** $d\mathbf{r} d\mathbf{v}$, **at a 6-D** point (\mathbf{r}, \mathbf{v}) , is:

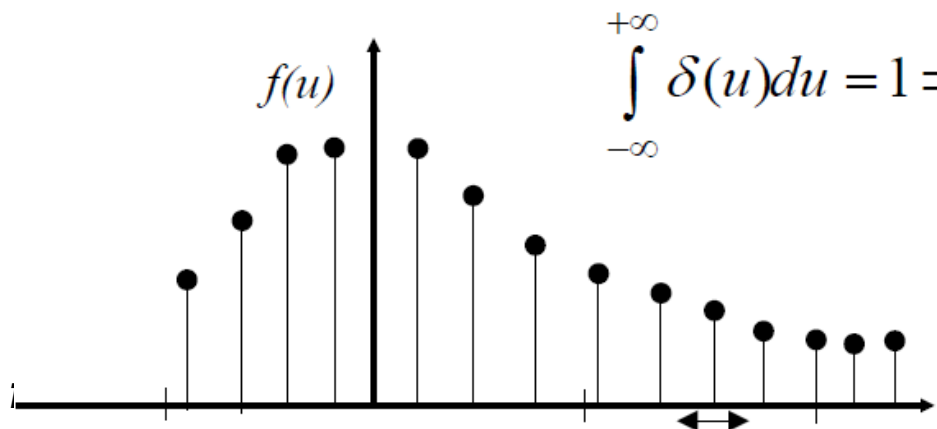
$$dN = f(\mathbf{r}, \mathbf{v}, t) d\mathbf{r} d\mathbf{v} = f(\mathbf{r}, \mathbf{v}, t) dr^3 dv^3$$

Sentido físico de la función de distribución delta

- Individual Velocity distributions (fixed position x or uniform spatial disposition):
- 1.- particle streams: uniform and two counter-streaming beams



- 2.- Distributed particles:



$$f(u, t) = \sum_i \delta(u - u_i) n_i \Rightarrow$$

$$\int_{-\infty}^{\infty} f(u, t) du = \sum_i n_i = n(t)$$

¿Units of Dirac-Delta, $n(t)$ and