Máster Universitario en Ingeniería Aeronáutica

The Space Environment

Collisional plasmas. Simple models

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Maretial y Página web de la Asignatura basada en la web personal del Prof. Dr. L. Conde:
https://plasmalab.aero.upm.es/~lcl/EntornoEspacial/

- As previously seen, if particle number does not change in a phase space volume element comoving with a "fluid" of point particles,a continuity equation holds for motion of *f* with fluid velocity ***U***

$$\frac{df}{dt} \equiv \frac{\partial f}{\partial t} + \frac{\partial}{\partial \mathbf{q}} \bullet \left(\boldsymbol{U}_q \, f \right) = \frac{\partial f}{\partial t} + div\left(\boldsymbol{U}_q \, f \right) = 0 \; ; \quad \boldsymbol{U}_q = (\dot{\mathbf{r}}, \dot{\mathbf{v}})$$

- However, If particle number *f d***q** *varies* (***instantaneous velocity changes ¡¡ they should seem to appear or dissapear from a small velocity volume, for a fixed r***) in a 6-D phase space elemental volume, for instance, due to particles interactions *in a shorter scale (inside a Debye length)* , the time rate of change of *f* is not zero : (Collisional **Boltzmann Equation**)

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \cdot (\mathbf{v} \, f) + \frac{\partial}{\partial \mathbf{v}} \cdot \left(\frac{\boldsymbol{F}}{m} f \right) = \left(\frac{\partial f}{\partial t} \right)_{coll} \neq 0 \quad ; \quad f \equiv f_\alpha , \;\; \alpha = e, i, a$$

- The *r.h.s*. means a balance of s**ource-sinks** of particles (density) in the volume element: **collision** may effectivily produce this effect, understood as if a particle with velocity **v** dissapears and another one appears with velocity **v'** (due to electromagentic field actions at microscopic level, even in a Debye Length range)

BASIC KEY IDEA It is consider now that there are collisions involved, introducing interaction forces that cause, even within a Debye sphere, an (almost) instantaneous change in the velocity of a particle test.

IN PRACTICE: The test particle with velocity v disappears from the element of volume and it appears as another particle of velocity v'

- If particle number $f\, d\mathbf{q}$ varies (***instantaneous velocity changes ¡¡***) in a 6-D phase space elemental volume, for instance, due **to particles interactions** in a shorter scale (inside a Debye length) , the (local co-moving) time rate of change of f is not zero :

KEY: now a cross the boundaries of an element of volume, measured in the particle fluid frame, it seems as if particles appear / disappear giving a net non zero flow

$$\frac{\partial f}{\partial t}+\frac{\partial}{\partial \mathbf{r}}\cdot(\mathbf{v}\, f)+\frac{\partial}{\partial \mathbf{v}}\cdot\left(\frac{\mathbf{F}}{m}f\right)=\frac{df}{dt}=\left(\frac{\partial f}{\partial t}\right)_{coll}=C(f)=\sum_{\beta}C(f,f_{\beta});\quad f\equiv f_{\alpha}$$

- The collisional term involves **forces** giving (apparently) **discontinuous changes** in the density number of volume element of phase space (due to *relatively abrupt changes* of velocity)

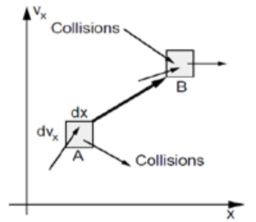
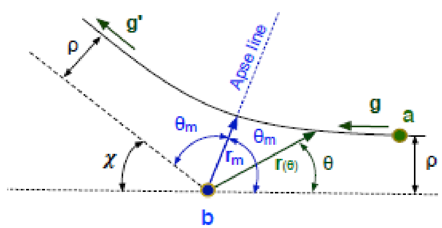
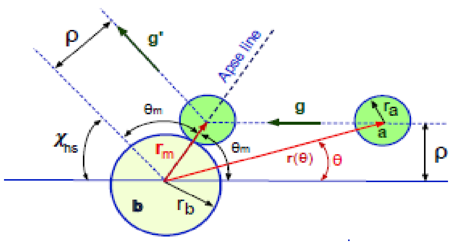


Figure 2.2: Evolution of phase space volume element under collisions



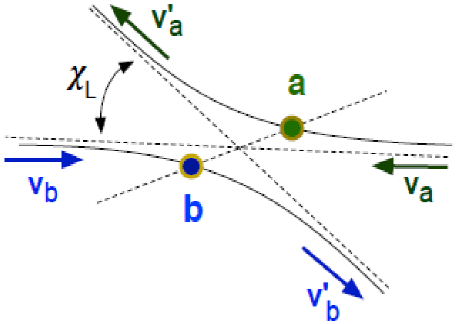


- The **cross-section of collisions between particle species** depends on the physics of the problem, as for hard-sphere collisions or for **Coulomb scattering** of charges (Modelos que implican tiempos distintos de interacción)

- Usually, the collision term is an **operator showing an integro-differential** form involving all f_α , **however, no exact one exists** but , for particular problems,
  Simplified collision terms *can be derived*

Collision term as a balance: Boltzmann Equation

For $f_\alpha(\mathbf{r}, \mathbf{v}, t)$ The Boltzmann operator applies under the ***binary collisions approximation (between only two particles):***



Binary **Coulombian collisions between charges** q_α and q_β mainly control particle densities, momentum fluxes and energy relaxation processes. Meaning:

$$\left(\frac{\partial f_\alpha}{\partial t}\right)_{col.} = C_s(f_\alpha) = \sum_\beta C(f_\alpha, f_\beta)$$

collisional time evolution rate of the α particles distribution function

Considers **all binary collision** processes with α ≠ β and α = β plasma charge species (interactions such as ***e-e, e-i, i-i, e-a …***)

A collision operator is constructed under the **binary** approximation **by studying the dynamics of the** *interaction (collision) between a single fixed test particle α and another field particle of the same or different kind β:*

$$\left(\frac{\partial f_\alpha}{\partial t}\right)_{\alpha\beta} dv_\alpha = \left[\left(\frac{\partial f_\alpha}{\partial t}\right)^{+}_{\alpha\beta} - \left(\frac{\partial f_\alpha}{\partial t}\right)^{-}_{\alpha\beta}\right] dv_\alpha$$

KEY: Collisional term is understood as a BALANCE of outgoing and incoming particles in a volume element

Boltzmann collision term as a balance *(intuitive approach)*

Balance: Equivalently, a number of *particles with* the velocities in $(v'_\alpha, v'_\alpha+dv'_\alpha)$ *are added (eliminated)* to the interval $(v_\alpha, v_\alpha+dv_\alpha)$, the net number of incoming (outgoing) particles in this interval is:

$$dn'_\alpha\big|_{\wedge col} = \left(\frac{\delta f'_\alpha}{\delta t}\right)_{col} d^3v'_\alpha\, \delta t\ ;\ \mathbf{g}' = \mathbf{v}'_\alpha - \mathbf{v}'_\beta, \quad \mathbf{g} = \mathbf{v}_\alpha - \mathbf{v}_\beta \quad elastic \rightarrow |\mathbf{g}'| = |\mathbf{g}|$$

$$\left(\frac{\partial f_\alpha}{\partial t}\right)^{+}_{\alpha,\beta} d\mathbf{v}_\alpha\, \delta t = f_\alpha(\mathbf{r}, \mathbf{v}'_\alpha, t) \times f_\beta(\mathbf{r}, \mathbf{v}'_\beta, t) \times (|\mathbf{g}'|\, \delta t) \times (b\, db\, d\theta)\, d\mathbf{v}'_\alpha\, d\mathbf{v}'_\beta$$

$$\left(\frac{\partial f_\alpha}{\partial t}\right)^{-}_{\alpha\beta} dv_\alpha\, \delta t = f_\alpha(r, v_\alpha, t)\, f_\beta(r, v_\beta, t) \times (|g|\, \delta t \times b\, db\, d\phi)\, dv_\alpha\, dv_\beta$$

b is an impart parameter, **g’** , the relative velocity after interaction. With the **collision differential cross section** that characterizes the mechanics of any (α, β) collision process (due to Coulomb forces ,mainly):

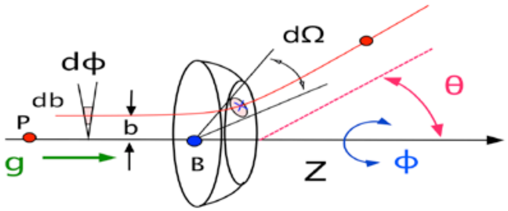
$$b\, db\, d\theta = \sigma_{\alpha,\beta}(g,\theta)\, d\theta\, d\phi = \sigma_{\alpha,\beta}(g,\theta)\, d\mathbf{\Omega}$$

The *Boltzmann collision operator* comes from **the balance** of incoming-outgoing particles due to (α, β) collisions

$$C(f_\alpha, f_\beta)\, d\mathbf{v}_\alpha = (f'_\alpha\, f'_\beta\, |\mathbf{g}'|\, d\mathbf{v}'_\alpha\, d\mathbf{v}'_\beta - f_\alpha\, f_\beta\, |\mathbf{g}|\, d\mathbf{v}_\alpha\, d\mathbf{v}_\beta)\, \sigma_{\alpha,\beta}(g,\theta)\, d\mathbf{\Omega}$$

For two colliding particles of species **(α, β), collisions term properties:**

Proportional to* number of encounters *(in – out) and relative velocities-
Physics enters in the integral cross-section computation (Mechanics)
Solid angle accounts all range of possible collisions
Neglects actions of other particles or collective forces: operates inside a Debye range extend.



The Boltzmann Equation (elastic collisions) See Ref 1.3 and Ref 2,1

- Resume and simplifications: If for the microscopic scale, between particles *binary collisions* are **elastic** $|\mathbf{g}| = |\mathbf{v}_\alpha - \mathbf{v}_\beta| = |\mathbf{v}'_\alpha - \mathbf{v}'_\beta| = |\mathbf{g}'|$

$$d\mathbf{v}_\alpha \, d\mathbf{v}_\beta = d\mathbf{v}'_\alpha \, d\mathbf{v}'_\beta$$

a simplification of the Boltzmann operator applies, giving:

$$C(f_\alpha, f_\beta) = \int \int (f'_\alpha \, f'_\beta - f_\alpha \, f_\beta) \, |\mathbf{g}| \, \sigma_{\alpha,\beta}(g, , \theta) \, d\Omega \, d\mathbf{v}_\beta$$

where the cross section σ depends on the particles properties , the nature of the interaction forces (microscopic fluctuating fields in fact) **and geometrical parameters.**

Also: the cross section is a function of **|g|,** as a first approximation chosen as constant**.**

- The **collective effects** (out of the Debye sphere) are accounted in the force **F** in *the term* (***F*** *f*(**r**,**v**,*t*) / *m*) of the kinetic equation, where ***F*** comes from the average mesoscopic electromagnetic fields obtained with the ***Maxwell*** *equations* with charge and current densities.
- The collisional effects also modifies plasma evolution entering in the **collective effects** in a larger time and spatial scales.

Steady state solution: The Maxwellian Distribution

1) From the **simplified Boltzmann** collision (bilinear) operator, for homogeneous distribution, and stationary solution exist:

$$C(f_\alpha, f_\beta) = \int\int (f'_\alpha f'_\beta - f_\alpha f_\beta)\, |\mathbf{g}|\, \sigma_{\alpha,\beta}(g,,\theta)\, d\Omega\, d\mathbf{v}_\beta$$

Relaxation to an equilibrium, contrary to Vlasov model:

2) Under **spatially homogeneous conditions** the **stationary solution** of the (any) Boltzmann equation is (always ¡¡) of the form: (unique solution)

$$f'_\alpha f'_\alpha - f_\alpha f_\alpha = 0 \Rightarrow f_\alpha = C_0 \exp(-a v^2 - \mathbf{b}\cdot\mathbf{v}) = K \exp\left(-a\,|\mathbf{v}-\mathbf{w}|^2\right)$$

Moreover: ***this solution maximizes the entropy of the system ¡ (collisions lead to irreversibility)***

It s easy to directly Identify the parameters *K*, *a* and **w** if the distribution function corresponds to one plasma species Maxwellian ensemble of density *n*, temperature *T* and average (flow).velocity ***u*.**

Notes: The stationary solution is a Gaussian in velocity, independent of the physics involved in the collisional process (effective section) valid for neutral gas and for plasmas in thermodynamic equilibrium, despite the disparity of phenomena involved and different scales.

The numbers of occurrences (encounters measured before and after interaction) are weighted by an effective “cross-section” and by the value of the “relative velocity”. Check the physical dimensions of the term C.

Collisional regime

- The Boltzmann (collisional) equation for distribution functions depends on the form of the collisional term $C(f)$

$$\frac{\partial f_\alpha}{\partial t} + \nabla_r \cdot (\boldsymbol{v} f_\alpha) + \nabla_v \cdot (\frac{\boldsymbol{F}}{m_\alpha} f_\alpha) = C(f_\alpha) \qquad f_\alpha(\mathbf{v}, \mathbf{r}, t) \qquad \alpha = e, i, a$$

- **F** accounts for **collective effects** (including self-consistent electromagnetic fields) and *C(f)* can be simplified by using somewhat simple models.

- A (non-magnetized) plasma is said highly "collisional" if the typical collision frequency (number of collisions in a second)

$$\nu_\alpha = \sum_\beta \nu_{\alpha\beta} \quad ; \; \alpha, \beta = e, i, a$$

for a α species is greater than the corresponding plasma frequency (*kinetic effects dominate locally over collective effects*)

$$\nu_\alpha \gg \omega_p \equiv \begin{matrix} (n_e e^2/m_e \varepsilon_0)^{1/2} \\ (n_i (Ze)^2/m_i \varepsilon_0)^{1/2} \end{matrix}$$

- In the opposite case, collisions may be ignored (as in most plasmas in the space enviroment ¡¡¡) at least in a time and a spatial scale, collective plasma behavior dominates, controlled by mesoscopic fields
  - However:***Thermalization and relaxation*** *to the thermodynamic equilibrium state requires of* ***collisional processes*** *to be reached* ¡

- ***LOCAL THEMODYNAMICAL EQUILIBRIUM can be considered as an approximation***

Approximate collision terms

If a “slightly” anisotropic plasma is close to the thermal equilibrium, i.e. ,
close to the **local equilibrium Maxwellian** ***f*** of mean velocity **u** and temperature *T*:

$$f_0(\mathbf{r},\mathbf{v}) = f_M(\mathbf{v}\,|\,\mathbf{u}(\mathbf{r}),T(\mathbf{r})) = n(\mathbf{r})\left(\frac{m}{2\pi k_B T(\mathbf{r})}\right)^{3/2} \exp\left(-\frac{m\,|\,\mathbf{v}-\mathbf{u}(\mathbf{r})\,|^2}{2k_B T(\mathbf{r})}\right)$$

At a given position of configuration space ***r***, it can be assumed that particles obey a local Maxwellian equilibrium distribution : **Local Thermodinamic Equlibrium** LTE

- The *effects of binary collisions* can be modeled by means of a collisional effective and almost constant collisional **frequency** to evaluate **the rates of loss and gain** of particles in the Boltzmann derivation introduced above.

BGK model

Simple BGK model, called relaxation model, approximates the collision operator as follows:

Ref. Bhatnagar, P.L., Gross, E.P. and Krook, M. Phys. Rev. ***94, 511****(1954)*

- For a single species plasma (*e.g.* electrons in a neutralizing background of ions at rest (Dirac delta distrib.) , a Lorentz electron gas):

$$\left(\frac{\partial f}{\partial t}\right)_c = -\nu_c (f - f_0)$$

Rate for loss of particles: $-\nu_c f$

Rate for gain of particles: $+\nu_c f_0$

$(\nu_c = \nu_c(|\mathbf{v}|)$ *is a measure for a collision frequency*)

- This $C(f)$ tends to equilibrate the difference between the distribution function and the Maxwellian distribution. Extension to several species is straightforward (see NRL Formulary)

The system characteristic **relaxation time** is given by the *inverse of the frequency* ν_c, this can be seen from the (free-force and spatially uniform) solution for constant frequency ν_c :

$$f(\mathbf{v},t) = f_0 + \left(f(\mathbf{v},0) - f_0\right)\exp(-\nu_c t) \;\; ; \;\; \nu_c = 1/\tau_r$$

1) Verify that **conservation properties** of this operator are satisfied by an appropriate election of the Maxwellian parameters for constant frequency (prove that the five moments n, **u** and T are constant)

- The distribution *fo* can be a **local *drifting* Maxwellian (it may be anisotropic)** if the system is *slightly departed from the equilibrium* distribution, this means that

$$|\mathbf{u}(\boldsymbol{r})| \ll \sqrt{k_B T / m}$$

holds for any **r** and assuming that collision frequency is of the order of plasma frequency.

- The model is **useful** due to its simplicity, **HOWEVER** it gives non-realistic **identical relaxation times** for all the distribution moments (density, momentum, energy…fluid properties)

- **APPLICATIONS:**

- It can be used to introduce the method to compute the **transport coefficients** (as plasma conductivities) by using the method of **small perturbations**.

The so-called *transport coefficients* relate the fluxes (of mass, charge and heat) with the generalized “thermodynamic forces” mainly, the electric field **E** and temperature (or density) gradients as: (flujo de calor –transporte de energía cinética- y corriente –trasnporte de carga- para ecuaciones de fluido)

$$\mathbf{q} = -\kappa\,\nabla T - \beta\mathbf{E}\;,\quad J = \sigma\,\mathbf{E} + \alpha\,\nabla T$$

Application to transport: Perturbation method. Example

A method to compute the **transport coefficients** (as plasma conductivities) operates as follows: by **linearizing** this Eq. assuming **a small deviation f_1 from the isotropic f_0** steady state distribution with homogeneous density n and temperature T :

$$f(\mathbf{r},\mathbf{v},t) \simeq f_0(v) + f_1(\mathbf{r},\mathbf{v},t) \quad ; \quad |f_1| \ll f_0$$

Example: **Electrical conductivity**. Assuming spatially uniform f_1, T and n, in a non-magnetized plasma (single species of **charge** Ze) in a ***small*** uniform field **E:**

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}} + \frac{q\mathbf{E}}{m}\cdot\frac{\partial}{\partial \mathbf{v}}\right)\underbrace{\left(f_0(v) + f_1(\mathbf{v})\right)}_{f} \simeq 0 + \frac{eZ\mathbf{E}}{m}\cdot\frac{\partial f_0}{\partial \mathbf{v}} = -\nu_c(f_1 + f_0 - f_0), gives\ f_1 \ (if\ \left|\mathbf{E}\ f_1\right| \to 0)$$

$$\sigma\mathbf{E} = \mathbf{j} = qn\mathbf{u} = Ze\int \mathbf{v} f\, d\mathbf{v} = Ze\int \mathbf{v} f_1 d\mathbf{v} = Ze\frac{Ze\mathbf{E}}{m\nu_c}n \quad , \ pues \int \mathbf{v} f_0 d\mathbf{v} = n\mathbf{u} = 0$$

$$\sigma = Z^2\frac{e^2 n}{m\nu_c} \ ; \ In\ general: \mathbf{j} = \sum_\alpha q_\alpha n_\alpha \mathbf{u}_\alpha \Rightarrow \sigma = \sum_\alpha \frac{Z_\alpha{}^2 e^2 n_\alpha}{m_\alpha \nu_{c\,\alpha}}$$

As greater the collision frequency is, the plasma is less conductive (is it real? The collision frequency rwould be proportional to 1/n ¡¡¡¡¡ . Really: This conductivity tends to a saturation value for large density n).

The heat flux **q** would lead to the thermoelectrical coefficient β:

$$\mathbf{q} = \int \frac{1}{2} m v^2\ \mathbf{v}\ f\, d\mathbf{v} = \int \frac{1}{2} m v^2\ \mathbf{v}\ f_1\, d\mathbf{v} = -\beta\mathbf{E}$$

Practica: Un ejemplo de término collisional (de Krook, BGK)

Problem 2) Consider the space 1-D homogeneous distribution governed by :

$$\frac{\partial}{\partial t} f(v,t) + a\frac{\partial}{\partial v} f = -\nu\left(f - f_0(v)\right) \;;\; (a\ ,\nu\ are\ \text{constants})$$

Where f_0 is a Maxwellian distribution of zero mean velocity and constant density n_0 and temperature T_0, the acceleration a is constant
Discuss the meaning of each term.

a) Verify that there is a stationary solution *f(v)*. Does this steady state solution depend on the initial function *f(v,0)*? ¿what boundary conditions should satisfied f?

b) Find the transient and steady solutions for a=0 and for arbitrary *f(v,0)*.

c) Compute for this solution *n(t)*, *u(t)* and *T(t)*.

$$Sol.\ f(v,t) = f_0(v) + (f(v,0) - f_0(v))e^{-\nu t} \rightarrow$$
$$n(t) = n_0(1 - e^{-\nu t}) + n(0)e^{-\nu t}$$
$$u(t) = n(0)u(0)e^{-\nu t} / n(t) \quad \text{and} \quad T(t) = \ldots$$

4) Consider the 1D kinetic equation in the convection-diffusion form:

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial v}\left[-\gamma(v-u) - \frac{\partial}{\partial v}D\right] f(v,t) \equiv -\frac{\partial j}{\partial v}$$

where

$$\int_{v=-\infty}^{\infty} f(v,t)dv = n(t), \quad n(t)u(t) = \int_{v=-\infty}^{\infty} v\, f(v,t)dv$$

Assuming that *j (a density flow)*, f and its derivative with respecto to v vanish for large $|v|$,

a) Discuss the meaning of each term (dealing D and γ as contants) in order to consider this equation as a collision 1-D operator (**Fokker-Planck** equation).

b) Using integration by parts on both sides of the eq. verify that n(t) =n(0) and u(t) = u(0) are constant in time and find D to ensure that the temperature T is also a constant (collisional invariant).

c) Assume that a stationary solution in the form

$$f_s(v) = C\exp(-a(v-u)^2)$$

exits and find it.

c) Find a simple extension of this collision term for the 3-D velocity distribution function.

$$Sol.consider: \ \gamma = \nu_c \quad and \quad D = kT\nu_c / m$$

Important : Inelastic Collisions

- *Some **plasma processes** like **recombination, ionization, dissociation,** source-sink for particles or charge-exchange can be dealt as **effective inelastic collision** terms to be added to the elastic (scattering) collision operators (particles removed or added to a volume phase space)*

They can be modelled by **appropriate cross sections** or frequencies. For example:
charge-exchange term between ions *i* and neutrals *a*, is :

$$\left(\frac{\partial f_i}{\partial t}\right)_{c,ch} = \int d^3v' \left|\mathbf{v} - \mathbf{v}'\right| \left(\sigma_{i,a} f_a(\mathbf{v}) f_i(\mathbf{v}') - \sigma_{a,i} f_i(\mathbf{v}) f_a(\mathbf{v}') \right)$$

where the collision **cross sections** σ depend on the energy

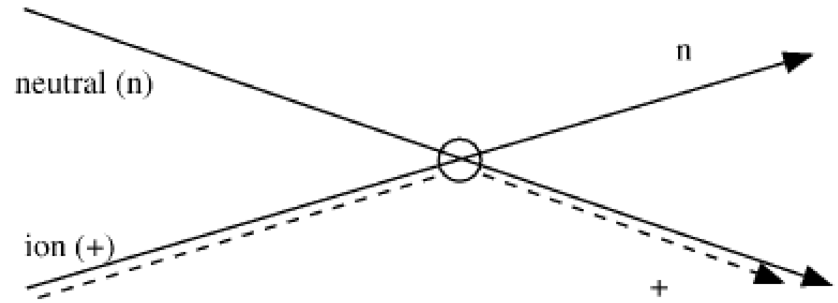


Figure 7.1: Charge exchange. The paths of two atoms are indicated by solid lines; the (+) label refers to an ionized atom, while (n) labels a neutral atom. Both paths are straight: the atoms do not suffer a mechanical collision. However, because of the charge-exchange event occurring at the intersection, the trajectory of *charge* changes direction, as indicated by the dashed lines. The result can be seen to resemble a mechanical collision, without charge exchange, between an ion and a neutral.

Generalization of Collision concept

Many other processes can be modelled as a Boltzmann term using appropriate effective collisional frequencies

• **Inelastic:** A fraction of the kinetic energy is used to alter the internal states and/or to produce new particles.
• **Superelastic:** The potential energy is transformed in kinetic energy; the kinetic energy of the system after the collision event is greater.
• **Radiative:** A fraction of energy is emitted and/or absorbed as electromagnetic radiation.
• **Charge exchange:** The electric charge state of colliding particles isinterchanged; an electric charge is transferred.

Production of charges by electron collisions

	Scheme	Process	Macroscopic effect
1	$e + A^+ \rightarrow e + A^+$	Elastic Coulomb collisions between electrons and ions	Transport and energy transfer in highly ionized plasmas.
2	$e + A \rightarrow e + A$	Elastic collision between electrons and neutrals	Electron transport and diffusion. Electron mobility.
3	$e + A \rightarrow e + A^*$	Excitation of neutrals by electron impact	Multiplication of metastable neutral atoms.
4	$e + AB \rightarrow e + AB^*$	Vibrational excitation	Energy transfer to vibrational levels of molecules.
5	$e + A \rightarrow 2e + A^+$	Electron impact ionization	Multiplication of ions and electrons from the ground state.

Recombination of charges by electron collisions

	Scheme	Process	Macroscopic effect
10	$e + AB \rightarrow e + A + B$	Molecule dissociation by electron impact	Production of neutral atom in molecular gases
11	$e + A^+ \rightarrow e + A$	De-excitation of neutrals (quenching)	Destruction of metastable neutral atoms
12	$2e + A^+ \rightarrow e + A^*$	Three body recombination	Only relevant in dense highly ionized plasmas
13	$e + A^+ \rightarrow h\nu + A$	Radiative recombination	

The **collision frequencies** (*averaged*) in Krook term can be estimated with more sophisticated models using methods of classical Mechanics for Coulomb forces particle interactions. For example using a *Fokker-Planck collisional model* the **relaxation times depend on the collision times** (see Braginskii fluid equations, **λ**=ln ***Λ***≃15, *T* en eV, cgs units)

$$\tau_e = \frac{3\sqrt{m_e}(kT_e)^{3/2}}{4\sqrt{2\pi}\, n\lambda e^4} = 3.44\times 10^5 \frac{T_e{}^{3/2}}{n\lambda}\,\mathrm{sec.}$$

$$\tau_i = \frac{3\sqrt{m_i}(kT_i)^{3/2}}{4\sqrt{\pi} n\, \lambda e^4} = 2.09\times 10^7 \frac{T_i{}^{3/2}}{n\lambda}\mu^{1/2}\,\mathrm{sec.}$$

electron collision rate	$\nu_e = 2.91\times 10^{-6} n_e \ln\Lambda T_e{}^{-3/2}\,\mathrm{sec}^{-1}$
ion collision rate	$\nu_i = 4.80\times 10^{-8} Z^4 \mu^{-1/2} n_i \ln\Lambda T_i{}^{-3/2}\,\mathrm{sec}^{-1}$

With this, It can be proved that **energy and momentum transfers** have different relaxation times

The (anisotropic) conductivity (parallel and transversal to a field B) becomes **independent of the density**:

as electron density increases, ion density also increases, as the collisional processes, this compensates the electron population effect

$$\sigma_{||} = 1.96\sigma_\perp;\ \sigma_\perp = ne^2\tau_e/m_e$$