Máster Universitario en Ingeniería Aeronáutica

The Space Environment

Plasmas as Fluids. Characterizing Parameters

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Maretial y Página web de la Asignatura basada en la web personal del Prof. Dr. L. Conde:
https://plasmalab.aero.upm.es/~lcl/EntornoEspacial/

From kinetic to fluid

The Boltzmann equation provides the distribution function motion for each plasma species, with appropriate collision terms, as

$$\frac{\partial f_\alpha}{\partial t} + \nabla_{\boldsymbol{r}} \cdot (\boldsymbol{v} f_\alpha) + \nabla_{\boldsymbol{v}} \cdot \left(\frac{\boldsymbol{F}}{m_\alpha} f_\alpha\right) = C(f_\alpha) \qquad f_\alpha(\boldsymbol{v}, \boldsymbol{r}, t) \quad \alpha = e, i, a$$

but the distribution function provides too many details, only the charge density ρ_C (r, t) and the electric current density J_C (r, t), (with energy transfers) are physically important ¡¡¡

$$\rho_c(\boldsymbol{r},t) = \sum_\alpha \rho_{e\alpha} = \sum_\alpha q_\alpha n_\alpha(\boldsymbol{r},t) \qquad \boldsymbol{J}_c(\boldsymbol{r}, t) = \sum_\alpha \boldsymbol{J}_{e\alpha} = \sum_\alpha q_\alpha n_\alpha \boldsymbol{u}_\alpha = q_\alpha \int_{-\infty}^{+\infty} f_\alpha(\boldsymbol{v}, \boldsymbol{r}, t)\, \boldsymbol{v}\, d\boldsymbol{v}$$

for Maxwell equations,***(ONLY TWO MOMENTS OF f)*** giving rise to the coupling of plasma dynamics and macroscopic (out-Debye-sphere) fields :

$$\nabla \cdot \boldsymbol{E} = \frac{\rho_c}{\varepsilon_o} \qquad \nabla \wedge \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{B} = 0 \qquad \nabla \wedge \boldsymbol{B} = \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} + \frac{\boldsymbol{J}_c}{\varepsilon_o c^2}$$

Using both **meaningful averages** and Maxwell´s Eq. all plasma properties would be calculated from ***f***, this seems to be a **well posed problem**, with a **closed system of equations**, **BUT**…

extremely difficult to solve, even for C(f)= 0 !! (Vlasov Eq.) **Why not use only density and fluxes (first moments of *f*) ? Is it possible to obtain motion equations for plasma species density, average fluid velocity and the system temperature, pressure, energy?**

YES , it can be done.

Derivation of fluid equations

- **A general distribution moment (average of a power of velocity component) or any average macroscopic evolution equation (transport equation) , can be easily obtained. For any scalar function H(v)**

$$\langle H \rangle(r,t) = \frac{1}{n_\alpha(\mathbf{r},t)} \int H(\mathbf{v}) f_\alpha(\mathbf{r},\mathbf{v},t) \; d\mathbf{v} < \infty, \;\; H = 1, v_x, v^2, v_x v_y, \ldots$$

- Multiplying by *H* both sides of the kinetic equation and integrating over velocity space :

$$\int d\mathbf{v}\left[\frac{\partial f_\alpha}{\partial t} H + \frac{\partial}{\partial \mathbf{r}}\cdot(\mathbf{v}\; f_\alpha \; H) + \frac{q_\alpha}{m_\beta} H \frac{\partial}{\partial \mathbf{v}}\cdot(\mathbf{E}+\mathbf{v}\times\mathbf{B})\; f_\alpha\right] = \int d\mathbf{v} H\left(\frac{\partial f_\alpha}{\partial t}\right)_c$$

with Gauss Theorem for the velocity divergence term,

$$\int d\mathbf{v}\left[H \frac{\partial}{\partial \mathbf{v}}\cdot(\mathbf{a}\; f)_\alpha\right] = \int d\mathbf{v} \frac{\partial}{\partial \mathbf{v}}\cdot[\mathbf{a}\, H\; f_\alpha] - \int d\mathbf{v}\; f_\alpha \mathbf{a}\cdot\frac{\partial H}{\partial \mathbf{v}}$$

with $\int d\mathbf{v} \frac{\partial}{\partial \mathbf{v}}\cdot[\mathbf{a}\, H\; f_\alpha] = \oint H\; f\; d\mathbf{S}_v \cdot \mathbf{a}_\alpha = 0$, making de surface $S_v = 4\pi\, v^2 \longrightarrow \infty$

$$\int d\mathbf{v}\left[H \frac{\partial}{\partial \mathbf{v}}\cdot(\mathbf{a}\; f_\alpha)\right] = -\int d\mathbf{v}\; f_\alpha\, \mathbf{a}\cdot\frac{\partial H}{\partial \mathbf{v}} = -n_\alpha\left\langle \frac{q_\alpha}{m_\alpha}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\frac{\partial H}{\partial \mathbf{v}}\right\rangle$$

Take CARE: assuming that $|v\, H\, f|$ decays to zero faster than $1/v^3$

the final time evolution of the H average is as follows :

Moments time evolution

- Finally, an effective and very general fluid equation is !!!!

$$\frac{\partial}{\partial t} n_\alpha \langle H \rangle + \frac{\partial}{\partial \mathbf{r}} \bullet \left[n_\alpha \langle \mathbf{v}\ H \rangle \right] - \frac{q_\alpha}{m_\alpha} n_\alpha \left\langle (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial H}{\partial \mathbf{v}} \right\rangle =$$

$$\int d\mathbf{v} \left[H \left(\frac{\partial f_\alpha}{\partial t} \right)_c \right] \quad ; \text{ for } \quad H = 1,\ v_i,\ \ v_i v_j,\ \ v_i v_j v_k,\ \ldots\ (i, j, k = x, y, z)$$

- A set of *scalar, vector or tensor equations* can be constructed from this. For example, without collision terms, for the zero- order moment H=1, we obtain the **continuity** equation for particle density, if the collision term is conservative (elastic):

$$\frac{\partial}{\partial t} n_\alpha + \frac{\partial}{\partial \mathbf{r}} \cdot [n_\alpha \boldsymbol{u}] = \int d\mathbf{v} \left(\frac{\partial f_\alpha}{\partial t} \right)_c = 0 \quad \text{(if no source-sink)}$$

and for $H = \mathbf{v}$, it gives the fluid equation for **momentum** transport:

$$\frac{\partial}{\partial t} n_\alpha \boldsymbol{u} + \frac{\partial}{\partial \mathbf{r}} \cdot \left[n_\alpha \langle \mathbf{v}\mathbf{v} \rangle \right] - \frac{q_\alpha}{m_\alpha} n_\alpha \langle (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \bullet \mathrm{I} \rangle = 0,\ si\ \mathbf{v} = \boldsymbol{w} + \boldsymbol{u}(\boldsymbol{r})$$

$$\frac{\partial}{\partial t} n_\alpha m_\alpha \boldsymbol{u} + \frac{\partial}{\partial \mathbf{r}} \cdot \left[m_\alpha n_\alpha \boldsymbol{u}\boldsymbol{u} + \underbrace{m_\alpha n_\alpha \langle \boldsymbol{w}\boldsymbol{w} \rangle}_{pressure-stress-tensor} \right] = n_\alpha\ q_\alpha\ (\mathbf{E} + \boldsymbol{u} \times \mathbf{B})$$

and so on. E.g. the equation for **kinetic energy (2nd order)** involves pressure tensor components , **heat flux** (3rd order moment) , and the internal and electromagnetic energies. The velocity is usually decomposed into two parts: macroscopic **u** and thermal fluctuating **w** for calculations.

Moments time evolution hierarchy equations

- Note that **every moment depends on the evolution of the higher order one**: the density depends on **u**, the fluid velocity **u** depends on **u u** (energy-pressure) and so on.
- For non-zero collision effects, there can be also additional terms responsible of particle and/or energy transfers among all plasma species.

Remarks (**drawbacks** of the fluid description):

1.- Any <H> evolution depends on spatial variations of the higher order moment < v H > (this is it depends on, the **transport of H** property itself !!):

an unclosed hierarchy of equations appears.

2.- A **collisional model is still needed** to evaluate the second hand terms for every moment.

3.- Approximations are needed to close (cut off) the fluid equations hierarchy, for example:

a) Use only the **two first**, neglecting energy transfers and using a state equation (**adiabatic,** isobaric, etc.) as

$$P_e n_e^{-\gamma} = cte. \;\; . \;\; P_i = n_i k T_i \;\; ....$$

b) Use the **transport coefficients**, for example, the thermal conductivity to obtain heat flux (3rd order moment) as a function of the gradient of temperature (2nd order moment)

4.- It is **still necessary** to use distribution functions models (microscopic scales) to obtain the time rates of particle, momentum and energy transfers (collisional frequencies and relaxation times) as well as for computing the transport coefficients (conductivities, viscosity …)

5,. A **Closure** for fluid equations is neededit implies : new approximations for each plasma regime

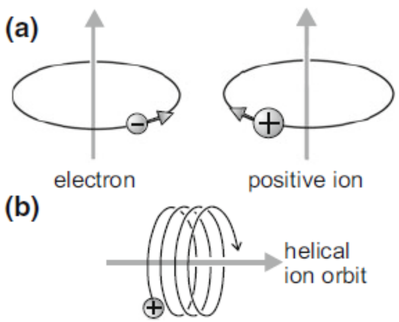
Non-Homogeneous and anisotropic plasmas are usual in Space

- FURTHERMORE: another problem emerges for magnetized plasmas, the system is strongly anisotropic.

- **Plasma spatial anisotropy emerges naturally when fields E and B appears. Fluid equations may be considered for plasma behavior along and transverse to the field (as in Solar flares or inner radiation Van Allen belts)**

E.g. In a strong magnetic field, plasma species can **be magnetized**, thus, the **pressure tensor becomes anisotropic, plasma is** far from equilibrium.
It is then convenient to distinguish two dynamics, **perpendicular and parallel** to ***B***,(very usual in space plasmas) with two temperatures, leading to an isotropic pressure tensor:

$$\boldsymbol{u} = u_{\parallel}\frac{\mathbf{B}}{B} + \boldsymbol{u}_{\perp} \quad , \quad \left\langle \frac{1}{2} m \left| \mathbf{v} - \boldsymbol{u} \right|^2 \right\rangle = \frac{3}{2} k_B T = k_B \left(T_{\perp} + \frac{1}{2} T_{\parallel} \right) \text{, isotropization relaxation: } \frac{dT_{\perp}}{dt} = -\frac{1}{2}\frac{dT_{\parallel}}{dt} = -\nu \left(T_{\perp} - T_{\parallel} \right)$$



$$\omega_c = \frac{|q|}{m} B_z$$

$$r_L = \frac{v_{\perp}}{\omega_c}$$

$$\langle \mathbf{ww} \rangle = \frac{kT_{\perp}}{m} \mathbf{1} + \frac{1}{m} k \left(T_{\parallel} - T_{\perp} \right) \frac{\mathbf{BB}}{B^2}$$

A plasma is magnetized if the gyro-radious (Larmor Radius r_L is small compared with size L of the system. The **gyro-frequency** (cyclotron) is associated to periodic motion el electrons and ions, usually electrons can be magnetized, but not the ions.

- **In weakly ionized plasma, electrons and ions coexists at quite different temperatures (e.g. 1eV electrons with ambient temperature ions) The relaxation to thermal equilibrium depends on a typical frequency**

Cold Plasma approximation: (plasma frío)

Neglects the thermal motions of the particles: the kinetic **pressure** tensor is taken **as zero**

For each species, N= ne and ni , with collision effects ***S*** as recombination (averaged) ($N = n$)

$$\frac{\partial N}{\partial t} + \nabla \cdot [N\mathbf{u}] = 0$$

$$mN\left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right] = qN(\mathbf{E} + \mathbf{u}\times\mathbf{B}) + \mathbf{S}_{ij}$$

$$\mathbf{S}_{ij} = -m\,N\,\nu_{\text{eff}}\,\mathbf{u}$$

Warm Plasma approximation: (plasma templado)

Considers ***scalar pressure*** only, and **additional state equations**. For N= *ne* and *ni* , with collision effects ***S*** (averaged) and e-g- $p=nKT.$

$$\frac{\partial N}{\partial t} + \nabla \cdot [N\mathbf{u}] = 0$$

$$mN\left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right] = -\nabla p + qN(\mathbf{E} + \mathbf{u}\times\mathbf{B}) + \mathbf{S}_{ij}$$

Two fluids model: for disparate electron and ion temperatures , each species **obeys independent** fluid **equations that can be coupled** by ***Maxwell Laws*** (by the fields **E** and **B** knowing charge and current densities) , charge-exchange or charge recombination terms (source-sink terms).

$$\frac{\partial n_e}{\partial t} + \nabla\cdot(n_e\mathbf{u}_e) = 0$$

$$\frac{\partial n_i}{\partial t} + \nabla\cdot(n_i\mathbf{u}_i) = 0$$

$$n_e m_e\left[\frac{\partial \mathbf{u}_e}{\partial t} + (\mathbf{u}_e\cdot\nabla)\mathbf{u}_e\right] = -n_e e\,(\mathbf{E} + \mathbf{u}_e\times\mathbf{B}) - \nabla p_e$$

$$n_i m_i\left[\frac{\partial \mathbf{u}_i}{\partial t} + (\mathbf{u}_i\cdot\nabla)\mathbf{u}_i\right] = +n_i e\,(\mathbf{E} + \mathbf{u}_i\times\mathbf{B}) - \nabla p_i\ .$$

$$\rho = n_i e - n_e e$$

$$\mathbf{j} = n_i e\,\mathbf{u}_i - n_e e\,\mathbf{u}_e$$

Magnetohydrodynamics (MHD) Single Fluid Equations

Single Fluid of MHD Equations (used in Solar Plasmas):

- Assume electrons and single-charged ions, **quasineutrality**
- **Magnetized,** ion gyroradius is small, (strong B) particle **mean-free paths are small** (high collisionality, the collisions have already done their work to quasi thermalize the whole system)¡¡
- **Neglect** heat fluxes, viscosities, electron-ion friction forces and electron-ion energy exchange terms (Te = Ti)
- Neglect electron mass, $m_e = 0$ and Introduce **mass density ρ**, fluid velocity **V = V** i, and total e-i pressure
- Coupling to approximate Maxwell`s Eqs. simple (but difficult to solve) MHD fluid equations

Ideal MHD predicts instabilities (as in fusion or solar flares plasmas) . In Gaussian units they are :

$$\frac{d\rho}{dt} + \rho \nabla \cdot \boldsymbol{V} = 0$$

$$\rho \frac{d\boldsymbol{V}}{dt} = \frac{1}{c}\boldsymbol{J} \times \boldsymbol{B} - \nabla p$$

$$\frac{dp}{dt} + \frac{5}{3} p \nabla \cdot \boldsymbol{V} = 0$$

$$d/dt \equiv \partial/\partial t + \boldsymbol{V} \cdot \nabla$$

$$\boldsymbol{E} + \frac{1}{c}\boldsymbol{V} \times \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{B} = \frac{4\pi}{c}\boldsymbol{J}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

Prob. Reescribir las ecauciones anterires en unidades del Sistema Internacional. Particulariza las ecuaciones si B y J son paralelos y E=0.

Basic parameters characterizing plasmas

- The main characteristic of a plasma is its **collective** response under any perturbation, neutral atoms or molecules, electrons and ions interact under both short.range and large-range (Coulombian) forces producing a collective behavior.
- Each **plasma species evolves in a particular way** giving rise to a system evolution that is usually far from thermodynamica equilibrium with no thermalization among several species.
- Microscopic (statistical and kinetic) description is neccesary as a basis to analyse collective phenomena that can appear thanks to the action of **mesoscopic and/or microscopic fields** (collisions)
- **Fluid description, of several or single species**, is useful in large systems time and space scales for which collisions have exerted its effects and mesoscopic fields and fluid magnitudes are enough to study the system
- The main magnitudes to clasify plasma regimes and the characteritic time and spatial scales are the nun,er density n and the temperatura T of electrons and ions.
- With n and T several **plasma parameters** can be defined to establish the time an space scale characterizing the plasma scenario and its mathematical/physical description, the main ones are:

Debye Length, Plasma Frequency,Plasma Number, Knudsen Number

Basic parameters characterizing plasmas

The damping and/or fluctuation of small densiies and electril local field from the equilibrium characterize the length and time scales of plasmas, See Ref 1,4

- Space fluctuations: *Debye length*
- Time fluctuations: *Plasma frequency*
- Number of charges: *Plasma parameter*

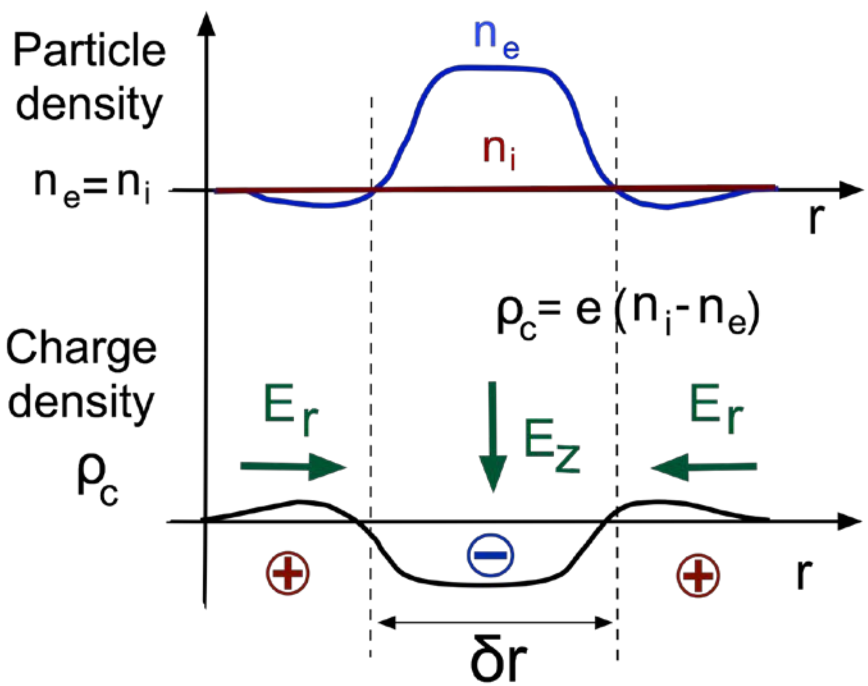
Small amplitude perturbations of plasma neutrality produce a local electric field.

$$\left\{ \begin{array}{l} n_\alpha = n_o + n_{\alpha 1} + O(n_\alpha^2) \\ \boldsymbol{E} = \boldsymbol{E}_o + \boldsymbol{E}_1 + O(\boldsymbol{E}^2) \end{array} \right\}$$

$$n_{e1} \neq n_{i1}$$

$$\nabla \cdot \boldsymbol{E}_1 = \frac{e}{\varepsilon}(n_{i1} - n_{e1})$$

$\boldsymbol{E}_o \simeq 0$ (plasma equilibrium)



The *Debye length* for ions and electrons of temperatures $k_B T_e$ a

$$\lambda_{De} = \sqrt{\frac{\varepsilon_o\, k_B T_e}{e^2\, n_{eo}}} \qquad \lambda_{Di} = \sqrt{\frac{\varepsilon_o\, k}{e^2}}$$

Usually, $k_B T_e \gg k_B T_i$ and $\lambda_{De} \gg \lambda_{Di}$

Time dependent fluctuations are governed by the *plas* *frequency* for ions and electrons are,

$$\omega_{pe} = \sqrt{\frac{e^2\, n_o}{\varepsilon_o\, m_e}} \qquad \omega_{pi} = \sqrt{\frac{e^2\, n_o}{\varepsilon_o\, m_i}}$$

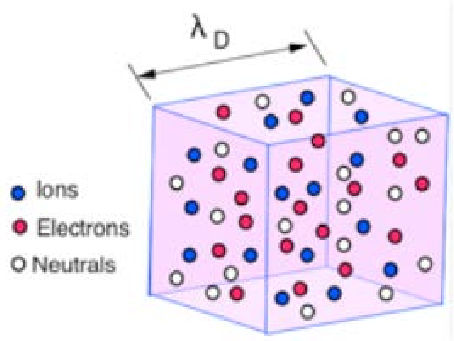
Basic parameters characterizing plasmas

For studyng a system as a plasma , the spatial size L and time observation sacale should satisfy the following relations , specially, the plasma number Nd (particles number of each species inside a debye sphere) has to be large:

$$\frac{\lambda_D}{L} << 1$$

$$\omega_p \tau >> 1$$

$$N_D = n_e \frac{4}{3} \pi \lambda_D^3 >> 1$$



An importatn parameter to specify if the regime is collionless or collisional is teh **Knudsen Number,** relevant in many analyses of plasma system and local spacecraf-plasma interactons

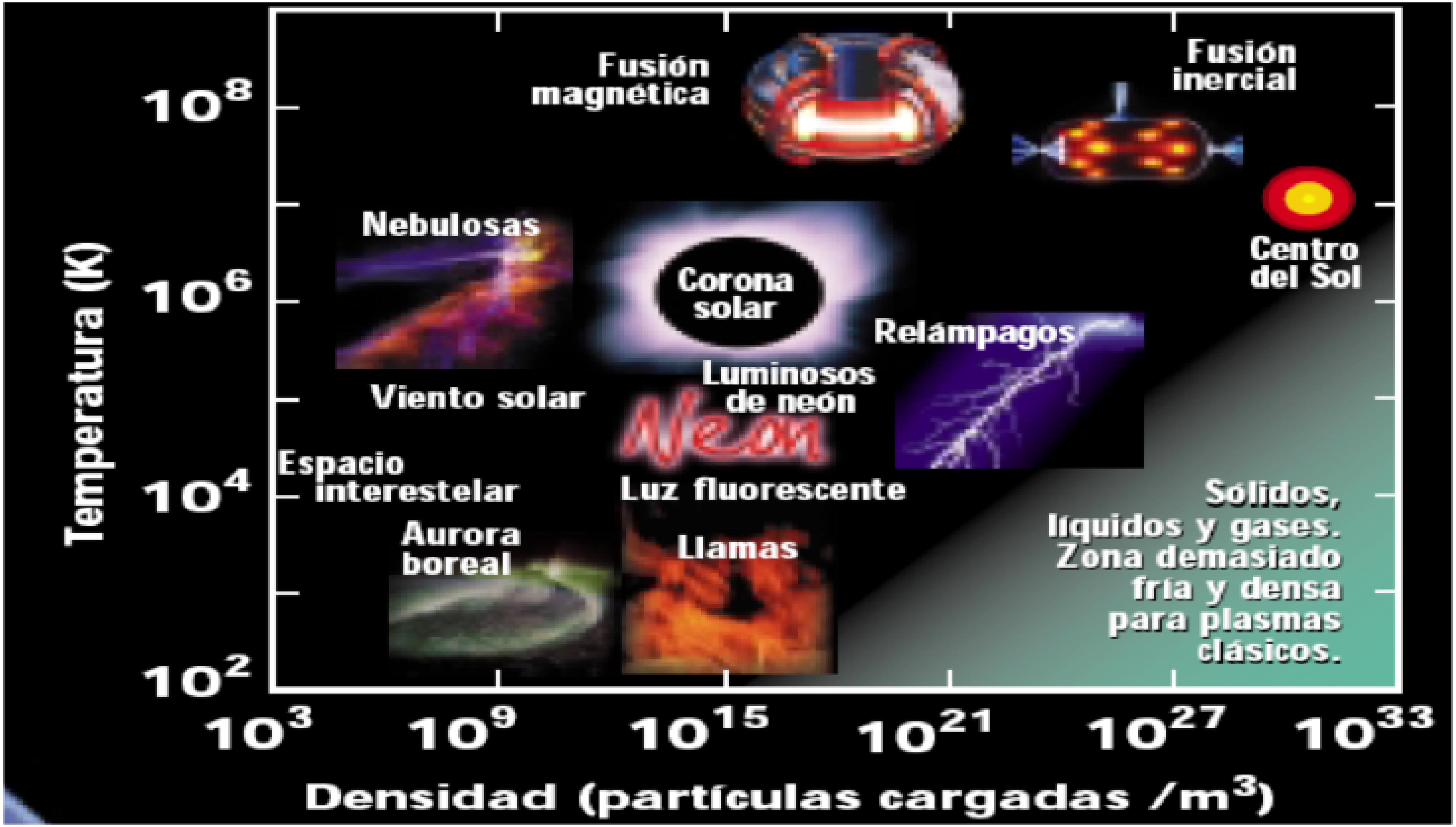
it compares the mean free path respect to the characteristic plasma scale (size L or, even, the Debye Lengnth):

$$Kn = \frac{\text{Collisional mean free path}}{\text{Characteristic length scale}} \qquad Kn = \frac{\lambda_c}{L}$$

- $Kn \ll 1$: Collisions are dominant, phenomena are described by conti
- .
- $Kn \gg 1$: Motion at molecular leve is described by probabilistic (kinet

Clasification

$$\frac{e}{k_B T} = \frac{1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 1,0} = 11.594,2 \simeq 11.600 \text{ K}$$



Data

Tabla1. Magnitudes Aproximadas en algunos plasmas tipicos

Tipo de Plasma	$n\ cm^{-3}$	$T\ eV$	$\omega_{pe}\ sec^{-1}$	$\lambda_D\ cm$	$n\lambda_D{}^3$	$\nu_{ei}\ sec^{-1}$
Gas Interstellar	1	1	6×10^4	7×10^2	4×10^8	7×10^{-5}
Nebulosa	10^3	1	2×10^6	20	10^7	6×10^{-2}
Corona Solar	10^9	10^2	2×10^9	2×10^{-1}	8×10^6	60
Plasma Caliente Difuso	10^{12}	10^2	6×10^{10}	7×10^{-3}	4×10^5	40
Atmosfera Solar,	10^{14}	1	6×10^{11}	7×10^{-5}	40	2×10^9
Descarga en Gases						
Plasma Tibios	10^{14}	10	6×10^{11}	2×10^{-4}	10^3	10^7
Plasma Caliente	10^{14}	10^2	6×10^{11}	7×10^{-4}	4×10^4	4×10^6
Plasma de fusion	10^{15}	10^4	2×10^{12}	2×10^{-3}	10^7	5×10^4
Theta pinch	10^{16}	10^2	6×10^{12}	7×10^{-5}	4×10^3	3×10^8
Plamsa Calientes Densos	10^{18}	10^2	6×10^{13}	7×10^{-6}	4×10^2	2×10^{10}
Laser Plasma	10^{20}	10^2	6×10^{14}	7×10^{-7}	40	2×10^{12}

Nota adicional. Ionización, Iones de carga múltiple.

- Un plasma en general está ***alejado del equilibrio*** termodinámico, solo aproxiamciónes.

- La relación de **Saha** da la fracción de átomos ionizados en función de la temperature:

$$\alpha = \frac{n_e\, n_i}{n_a} \approx 2.4\times 10^{21}\, T^{3/2} \exp(-E_I/k_B T)$$

$$T = 300\ K \qquad E_I = 15.8 \qquad (\text{Argon}) \qquad \alpha \sim 10^{-120}$$

- Densidades (numéricas) *n* y temperaturas *T* caracterizan el gas ionizado y el plasma

- EL plasma necesita **gran aporte de energía** para sostenerse (aumentar grado de ionización)

- El plasma es **cuasineutro**, mesoscópicamente, en promedio:

$$n_e \simeq Z\, n_i \rightarrow \rho_q = -e(n_e - Z\, n_i) \simeq 0$$

Pero esta neutralizad se viola localmente (microscópicamente) en las inmediaciones de cada carga .

$$\nabla \cdot \boldsymbol{E} = \frac{e}{\epsilon_o}(Z\ n_i - n_e)$$

A cierta distancia (Debey) de una carga *el campo local decae*, y prevalece la neutralidad.